Building up
Mathematics

Basic Education

6th Grade

$$\div 6$$

$$\div 2$$

$$\frac{18}{24}$$

$$\frac{\cdots}{12}$$

$$\div 2$$

$$\div 3$$

20.8%

16.7 %

37.5%

25%

National Textbook

NEW CURRICULA

Center for Educational Research and Development

Republic of Lebanon

Ministry of Education and Higher Education

BUILDING UP
MATHEMATICS

Basic Education

Grade Six

Center for Educational Research and Development

National
Textbook

New Curricula

General Coordinator
Victor Melhem

Language Reader
Ahmad Oweini
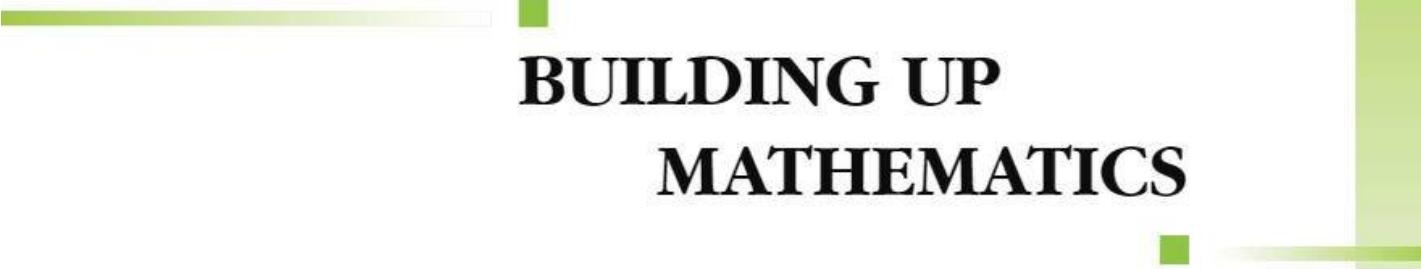
BUILDING UP
MATHEMATICS

Basic Education
Grade Six

Habib Abou Nader (Coordinator)
Saria Majzoub Daher

Kassem Beydoun
Maurice Charbel
Mirna Melhem
Nada Moubarak Feghali

The National Textbook Project

By issuing the textbooks for the third year of each educational cycle, the Educational Center for Research and Development will have completed the third and last installment of books called for by the New Curricula. We are placing these books in the hands of students with the great hope that we are moving, step by step, toward the goal of acquiring sound learning, using sophisticated educational means and up-to-date methodology that encourage and reinforce individual thinking and research, acquisition of skills, development of ethical and national attitudes, the feeling of national belonging as well as the feeling of belonging to humanity at large.

The on-going revolution in information, communication and educational-aids technology has undoubtedly limited the role of the textbook and lowered the rank it used so recently to occupy. However, in our society and in many other societies, the textbook remains the basic means of education, and it is our duty to exert our utmost effort and care to come up with the best product as to form and content. Yet we should not lose sight of the fact that the textbook is not sufficient by itself but should rather be used as a stepping stone to access other sources of information. What is important is to keep a clear vision and maintain the right course toward our objective. The means should not turn into the end and the student should always remain the focus of the learning/teaching process.

No one ignores or denies the fact that textbook writing requires very high academic and educational qualifications and very wide field experience. The authors committees undeniably possess such qualities. Yet the textbooks of the last two years contained some negative aspects. Such is the nature of human work, no matter how good the intentions or how great the effort extended. Here constructive criticism constitutes a real contribution to raising the standard of authorship, minimizing errors and filling gaps. We say that, with all due appreciation and respect to all those who have contributed to the success of this project.

The Educational Center for Research and Development is embarking this year on a process of evaluating the New Curricula and related textbooks, teacher training courses and student achievement. This is a natural and necessary step now that the new system has been put into effect. This process aims at identifying the curricular objectives that have been achieved as well as those that have not been achieved, with a view to proceeding with the positive aspects and correcting the negative ones.

As part of this correction process, we plan to review the versions that have been issued in order to secure good textbooks for our students, who always deserve the best.

March 13, 2000

President, Educational Center for
Research and Development

Nemer FRAYHA

Introduction

This book was designed to meet the needs of the sixth grade in basic education. Its purpose is to respond to the spirit of the new curriculum prescribed by the decree No. 10227 of 8 March 1997, and to complement the series entitled "Building up Mathematics" for the second stage in basic education.

Deriving its goals from educational research work based on formative and mathematical epistemology, the curriculum requires a particular teaching method that develops in the students a sense of initiative and freedom.

The method challenges the students with actual problems taken from real life and the environment urging them to conjecture, experiment and verify different solutions so as to gradually arrive at approaches that will later enable them to build their own models, as personal early initiatives of genuine mathematical models.

The activities presented in the various chapters constitute actual challenges to be met by the students. Students should address those challenges with independence and a sense of freedom in dealing with various data and premises so as to develop their skills and their ability to express themselves and to communicate.

To achieve the objectives set out at the beginning of each chapter, it is hoped that those who will use book as a teaching tool will observe to the following:

1. The order and sequence of the lessons;

2. The methodological structure of each lesson... "Getting familiar with my book".

3. The guidelines presented in the Teacher's manual.

Critical comments, inquiries and suggestions by users of the book are welcome and will be given the utmost attention.

The authors

USING THIS BOOK

A) To benefit from this book, you should:

1- Know how it is structured (succession of paragraphs).

2- Understand the role of each paragraph.

3- Complete the task set for each paragraph before moving to the next .

B) Structure of each chapter

Corresponding Tasks and Roles

Activities:

These are real and simple work situations that allow the learner to invest his knowledge in a new search while giving him a better grasp of the subject of the chapter. Failing to participate in this activity would be similar to missing the entire lesson.

Text:

This is the theoretical part that:

- is derived from the situation treated in *Activities*.
- makes possible the solution of all similar situations.
- constitutes the mathematical core of the chapter.
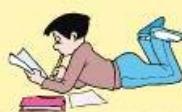
Focus:

This part summarizes the ideas and skills presented in the Text.

Exercises

These are designed for training in various situations, numerically or formally, to handle the new ideas of the chapter, to generalize them, and to become familiar with all sorts of relevant questions.

Self-evaluation:

Here the student can test his newly acquired knowledge.

Problems:

They re-examine the studied concepts along with other knowledge. This helps the student balance various types of learning.

Just for Fun:

These are thinking games whereby the student thinks freely as he enjoys the game. It does not really matter whether he finds the solution or not.

Table of contents

Order of operations

Objectives

At the end of this chapter, I will be able to perform calculations on positive numbers by applying the law of order of operations.

Activities

We use parenthesis ( ) to indicate the part of the expression that has to be simplified first, parentheses are always used in pairs facing each other. (...............)

a) In the first three basketball games this year, Zahi scored 21, 7 and 9 points. Sara scored 9, 7 and 21 points.
- Write an expression showing the total points of Zahi's score and Sara's score.
- How many points did Zahi score in the 2nd and 3rd game combined?
- How many points did Zahi score in the first game more than in the second and third together?
- Write an expression showing how many points Zahi scored in the 1st game more than in the 2nd and the 3rd game together by using the numbers 21, 9 and 7 (with the help of parentheses).

b) In the fourth game Zahi played 4 matches. In the 1st he scored 7 points. In the last 3 matches he scored 5 points per match. How many points did Zahi score in the 4th game?
- Sara and Zahi played 5 matches. Zahi scored 4 points in each and Sara scored 6 points in each. How many points did Sara score? Zahi score? Sara and Zahi score?
Write the sum of Sara's and Zahi's score in 2 different expressions.

c) In the last game, Zahi bought 53 bulbs to light the playground at night. He distributed the bulbs among 4 lines in such a way that the first line had 5 more bulbs than the other three.
What is the number of bulbs in each line?

d) Copy and complete:

$21 + 9 + 7 = ...$	$9 + 7 + 21 = ...$	$7 + 3 \times 5 = ...$	$7 + (3 \times 5) = ...$
$21 - 9 - 7 = ...$	$21 - (9 + 7) = ...$	$5 \times (4+6) = ...$	$(5 \times 4)+(5 \times 6)=....$

Compare the results obtained in column 1 with those of column 2 and column 3 with those of 4.
What can you deduce?

An expression which includes numbers and operations and no variables is called a numerical expression.

If we perform by varying the order of operations, we may get different answers.

Ex: 15 - 7 + 2. If we add first we get: 15 - 9 = 6. If we subtract first we get: 8 + 2 = 10. We obtain different answers. This means that we need to stick to one way to perform such operations so that everybody always gets the same answer.

A) Without parentheses:

Similarly in expressions in which multiplication and division appear. If we perform the operation in any order, we may get different answers each time.

- When multiplication and division are found in the same expression, they should be performed in the order by which the symbols appear from left to right.
- Multiplication and division should be done before addition or subtraction, when both operations appear in the same expression.

$7 \times 2 - 1$

$9 - 7 - 3$

Numerical expression

$12 \times (5 - 2)$

$48 \div 4 \div 3$

$5 + 7 + 3$

B) With parentheses:

Start with what is indicated within any parentheses then follow the above rules.

$$10 \times (48,5 - 13) + (9,1 + 0,9) \div 4$$
$$= 10 \times 35,5 \quad + \quad 10 \div 4$$
$$= \quad 355 \quad + \quad 2,5$$
$$= \quad 357,5$$

Examples

$$A = 13 - \underbrace{2 \times 3.5} + \underbrace{2.4 \div 3}$$
$$A = 13 - \underbrace{\quad 7 \quad} + 0.8$$
$$A = \quad 6 \quad + 0.8$$
$$A = 6.8$$

$$B = \underbrace{60 \div 12} \times 9 - 3.2$$
$$B = \underbrace{5 \times 9} - 3.2$$
$$B = 45 - 3.2$$
$$B = 41.8$$

Help me calculate numerical expressions.

Priority rule:

1- First, do what is indicated within parentheses.

2- Next, complete the indicated multiplication and divisions from left to right.

3- Finally, complete the indicated additions and subtractions from left to right.

Ex: $33 - (7 + 3) + (4 \times 11) - (54 \div 9)$

$$= 33 - 10 + 44 - 6$$

$$= 23 + 44 - 6$$

$$= 67 - 6$$

$$= 61.$$

Exercises

1- Copy and complete:

a) $47 + 18 - 13 - 11 + 9$

$= \ldots\ldots\ldots - 13 - 11 + 9$

$= \ldots\ldots\ldots\ldots - 11 + 9$

$= \ldots\ldots\ldots\ldots\ldots\ldots + 9$

$= 50$

b) $35 - 4 \times 3 + 24 \div 6$

$= 35 - \ldots\ldots + \ldots\ldots$

$= \ldots\ldots + 4$

$= 27$

c) $73 - (25 - 11)$

$= 73 - \ldots\ldots = \ldots\ldots\ldots\ldots$

d) $73 - 25 - 4$

$= \ldots\ldots - 4 = \ldots\ldots\ldots\ldots$

e) $48 \div (12 \div 4)$

$= 48 \div \ldots\ldots$

$= \ldots\ldots\ldots\ldots$

f) $48 \div 12 \div 4$

$= \ldots\ldots \div 4$

$= \ldots\ldots\ldots\ldots$

g) $7.2 \times (1,4 + 7,2)$

$= 7.2 \times \ldots\ldots = \ldots\ldots\ldots\ldots$

h) $(60 \div 12) - (1.3 \times 0.5)$

$= \ldots\ldots - \ldots\ldots$

$= \ldots\ldots\ldots\ldots$

i) $(7 \times 3) - (28 \div 7)$

$= \ldots\ldots - \ldots\ldots = \ldots\ldots\ldots\ldots$

j) $9 \times (18 \div 3)$

$= 9 \times \ldots\ldots = \ldots\ldots\ldots\ldots$

k) $9 \times (18 - 3) + 2(32 \div 8)$

$= 9 \times \ldots\ldots + 2 \times \ldots\ldots$

$= \ldots\ldots + \ldots\ldots = \ldots\ldots\ldots\ldots$

2- Seven boxes of chocolate cost 21000 LL. How much do three boxes cost?

3- Write true or false:

a)

$10 + 5 \times 7$ 105

b)

$38 \times 2 \div 19$ 4

c)

$28 - 276 \div 12$ 5

d)

$87 + 3 \times 4$ $57 + 33$

4- Use the calculator: You should determine whether your calculator follows the order of operation.

ex: Calculate: $7.2 + 32.1 \times 8$.

$\boxed{7} \to \boxed{.} \to \boxed{2} \to \boxed{+} \to \boxed{3} \to \boxed{2} \to \boxed{.} \to$

$\boxed{1} \to \boxed{X} \to \boxed{8} \to \boxed{=}$

- If your calculator displays 264 which is the correct answer, your calculator is programmed to follow the order of operation.

- If your calculator displays 314.4, you must always enter the operation in the correct order to get the correct answer.

$\boxed{3} \to \boxed{2} \to \boxed{.} \to \boxed{1} \to \boxed{X} \to \boxed{8} \to \boxed{+} \to$

$\boxed{7} \to \boxed{.} \to \boxed{2} \to \boxed{=}$

Use the calculator to find the value of the following expressions:

a- $14.5 \times (7.21 + 3.8)$

b- $140 \div 28 + 94.103$

c- $245 + 73 \times 104$

d- $270 + 71 \times 17$

e- $148 + 52 : 20$

f- $91.4 + 23.3 - 5.1 \times 3.41$.

5- Copy and complete:

a) $100 - 7 - 7 - 7 - 7 = 100 - 7 \times \ldots$

b) $10 + 13 + 13 + 13 = 10 + \ldots \times \ldots$

c) $500 \div 5 \div 5 \div 5 = 500 \div (5 \times \ldots)$

d) $13 \times 14 = (13 \times 7) + (13 \times \ldots)$.

6- Insert parentheses in the following expressions so that they all have the same answer:

a) $7 \times 3 + 9 - 2 =$

b) $7 \times 3 + 10 - 3 =$

c) $7 \times 5 + 7 \times 1 =$

d) $2 \times 7 \times 2 =$

e) $35 \div 5 \times 4 =$

7- Place parentheses to make the following expression true:

a) $14 + 73 \times 5 - 11 = 368$

b) $123 - 391 \div 23 + 104 = 210$

c) $1.36 \times 17 - 13 - 0.12 = 10$

d) $12 + 72 \div 9 - 1 = 19$.

8- Put a sign (operation) in each space below to make each of the following true:

a) $13.2 \ldots 7 \ldots 3 = 34.2$

b) $48 \ldots 4 \ldots 4 = 3$

c) $22 \ldots 3 \ldots 14 \ldots 28 = 51$

d) $13 \ldots 22.5 \ldots 17.2 \ldots 2.2 = 6.1$.

e) $\dfrac{7}{3} \ldots \dfrac{11}{21} \ldots \dfrac{1}{33} = \dfrac{1}{27}$.

9- Calculate A, B, C, D, E and F then match the equal expressions:

$A = 14.32 + (7 + 5.4) =$

$B = 14.32 + (7 - 5.4) =$

$C = (5.4 + 14.32) + 7 =$

$D = (14.32 - 7) + 5.4 =$

$E = (14.32 - 7) - 5.4 =$

$F = (14.32 + 7) - 5.4 =$

10- a) Perform the following operations:

$A = 21.1 + 2.51 - 17.21$

$B = 102 - 41 + 13.7$

$C = 108.3 + 211 - 291.83$.

b) Calculate:

$A \times B$; $A \times C$; $(A \times B) + (A \times C)$;

$A \times (B \times C)$.

11- Find the value of the following expressions:

a) $(24.2 \times 5.7) + (47.8 - 6.7) - (251.44 \div 2)$
$- (48.3 - 14.1)$

b) $(71.7 - 11.5) - (1.73 + 2.5) + (17.1 - 14.7) -$
(7.3×3).

12- Copy and compete:
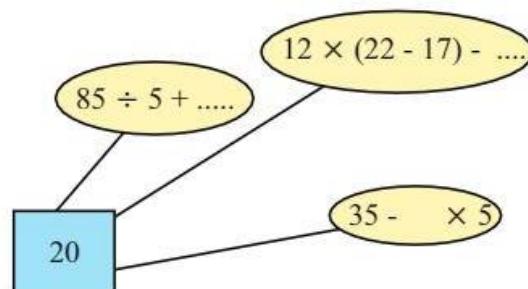
13- If 4 bottles of lemonade cost 4800 LL and 5 ice-cream cones 3750 LL, what is the total cost of 3 bottles of lemonade and 4 ice cream cones?

14- Maya is 9 years old. Her mother is one year less than four times Maya's age. What is mother's age?

15- Five dozens of eggs cost 12 000 LL. What is the cost of 18 eggs?

16- 2 km^2 of green field need 200 kg of fertilizer. How many kg of fertilizer is needed to service 8 km^2 of green field?

17- If Rola can run 15 m in 20 seconds, how far can she run, keeping the same speed, in 2 minutes?

Self-evaluation

1. **Answer by true or false and correct when it is false :**

   a) $(24 + 54) \times 6 = 468$

   b) $38.6 - 2 \times 7.6 - 15.2 = 23.4$

   c) $30 + 51 \div 3 = 81 \div 3 = 27$

   d) $15 + 2 \times 9.08 = 17 \times 9.08 = 154.36$.

2. **Put parentheses, if needed, to make the following expressions true:**

   a) $7 \times 2.6 + 3.4 = 42$;   b) $23 \times 72 \div 9 = 184$.

3. **Match the equal expressions:**

   a) $(6.7 \times 7) + 12.03$ •   •   a) 58.

   b) $48 + 30.7 - 20.6$ •   •   b) 58.93.

   c) $24 + (204 \div 6)$ •   •   c) 58.1 .

Problems

1- Mr. Rami bought 3 tables and 18 chairs for 153 000 LL. If the price of each chair was 6 000 LL,

a) what would be the price of the 3 tables? (in one step).

b) what would be the price of the 7 tables?

2- In a school library, 32 books were put on a shelf whose length was 120 cm the width of each book 4 cm.

How many more books can we put on this shelf (each book has the same width of 4 cm)?
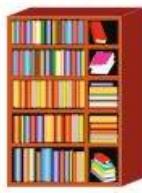
3- Nada is in the supermarket. She buys:

   3 jars of jam 1.5 kg each for 6 200 LL.
   3.5 kg of apple for 7 000 LL.
   400 g of cheese for 4 750 LL.
   2.5 kg of meat for 17 500 LL.

a) If Nada can only carry 8 kg how many kg are left for her sister to help a carry?

b) If Nada has only a 50 000 LL bill, how much money does she have?

4- The area of a strawberry garden is $256 m^2$. During winter the gardener covered the garden with plastic. On the first day he covered $48 \, m^2$ of the garden. The gardener divided the rest of his work over 4 days. How many square meters did he cover each day?

5- Rana bought 30 liters of gasoline for 575 LL. per liter. She also bought 3 kg of motor oil for 9.750 LL.

Find the total amount of money Rana paid in one step only.

15

6- Copy and complete:
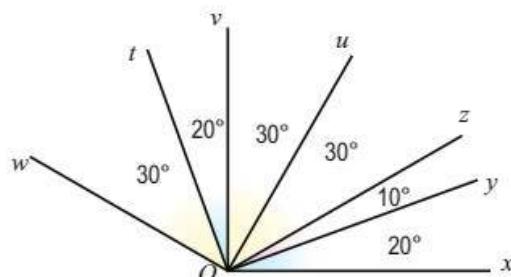
$$\widehat{xOw} = \boxed{} + 2 \times \boxed{} + 3 \times \boxed{}$$
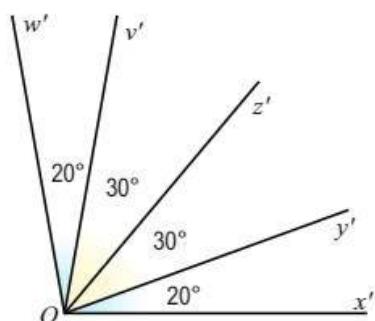
$$\widehat{x'O'w'} = 2 \times (\boxed{} + \boxed{})$$

7-

Restaurant menu		
Fries 2 000 L.L.	Yogurt 1 500 L.L.	Hamburger 2 250 L.L.
Fried chicken 2 750 L.L.	Hommos 1 300 L.L.	Chawarma 2 500 L.L.

What did these people order?
a) Rola : $(2 \times 2\,750) + 1\,500.$

b) Hazem : $1\,300 + (1\,500 + 2).$

c) Mr. Nader and his family

$(3 \times 2\,750) + (2 \times 2\,500) + 2\,250.$

8-

Price Tag		
T shirt 3 000 L.L.	Belt 2 500 L.L.	Jeans 11 000 L.L.
Skirt 5 000 L.L.	Socks 1 500 L.L.	Blouse 4 000 L.L.

Find the total amount that Zahi should pay if he buys: 3 T-shirts, 4 pairs of socks, 2 jeans, 1 pair of pants and 1 belt?

Add the numbers you see on any line: across, vertically or diagonally. The answer is 264. Now turn the page upside down and find the sum across, vertically or diagonally.

96	11	89	68
88	69	91	16
61	86	18	99
19	98	66	81
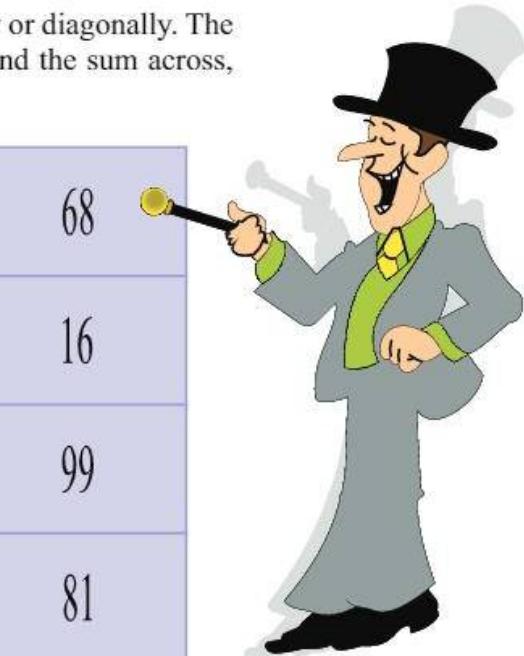
Development of numbers

Objectives

At the end of this chapter, I will be able to:

- Write and read a number; develop a number and find it starting from its development

Activities

The number of inhabitants of our world exceeds six billions.

The population of China represents $\frac{1}{6}$ of that of the whole population.

a) On your notebook, write in letters the number of inhabitants of each country:

LEBANON	JAPAN	BAHRAIN
3 895 000 habitants	123 100 325 habitants	628 573 habitants
SYRIA	CHINE	IRAQ
14 890 000 habitants	1 236 700 000 habitants	21 200 000 habitants
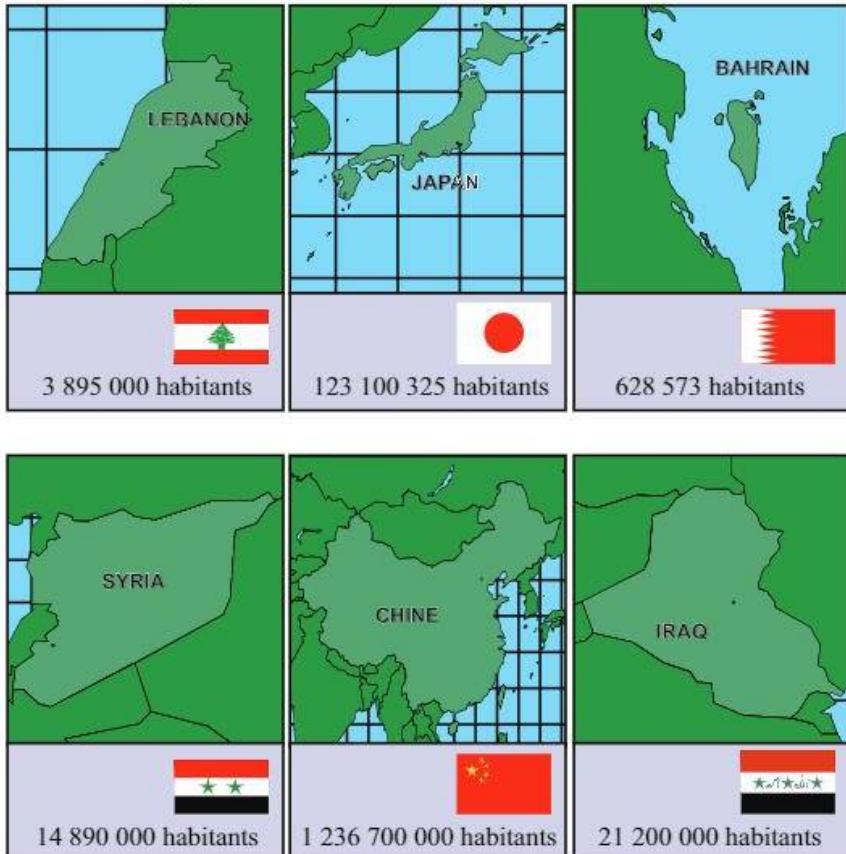
b) Observe the model and complete

3 895 000 = (3 × 1 000 000) + (8 × 100 000) + (9 × 10 000) + (5 × 1000)

1 236 700 000 = (1 × ---) + (   ) + (   ) + (   ) + (   ).

Text

twenty-four billion

five hundredninety-eight million

three hundred sixty thousands

A- • To write a greater number, we group the digits in threes.

• **Ex**: Write in standard form:

Twenty-two billion, five hundred ninety-eight million, three hundred sixty thousands.

B- The value of digit 6 is   60 000
The value of digit 3 is   300 000
The value of digit 8 is   8 000 000
The value of digit 9 is   90 000 000
The value of digit 5 is   500 000 000
The value of digit 4 is   4 000 000 000
The value of digit 2 is   20 000 000 000

So, the given number can be written in standard form:

24 598 360 000

and in expanded form:

(2 10 000 000 000) + (4 1 000 000 000) + (5 100 000 000) + (9 10 000 000) + (8 1 000 000) + (3 100 000) + (6 10 000) .

Focus

In a TV game show, Rami won the amount of 123 035 382 LL.
How would you write and read this number?
lThe place value chart helps us read and write large numbers.

Place Values											
Billions			**Millions**			**Thousands**			**Ones**		
c	d	u	c	d	u	c	d	u	c	d	u
		4	6	3	0	3	0	0	2	0	6

We can read and write large numbers represented in the chart in two different ways:

Read: 1- Short word form: 123 billions, 35 millions, 382 thousands.

2- Word form: One hundred twenty-three billions, thirty-five million, three hundred eighty-two thousands.

Write: 1- Standard form: 123 035 382 000.

2- Expanded form: $(1 \times 100\,000\,000\,000) + (2 \times 10\,000\,000\,000) +$
$(3 \times 1\,000\,000\,000) + (3 \times 10\,000\,000) + (5 \times 1\,000\,000) +$
$(3 \times 100\,000) + (8 \times 10\,000) + (2 \times 1000).$

We have two types of numbers:

1- Cardinal numbers: any of the numbers 1, 2, 3,.............

2- Ordinal numbers: related to position; first, second, third, ...

Exercises

1- Read each of the following numbers:

379 ; 7807 ; 28 483 ; 345 900 ;
3 648 788 ; 126 378 504 ; 5 780 538 419.

2- Read each of the following numbers:
One thousand; one million; one billion
Two million eight;
Five billion three hundred;
Fifteen millions six hundred eighty thousand;
Seven billions eight hundred five millions nine hundred eighty thousand three hundred.

3- Write in letters the following numbers:
9 678 ; 385 701 ; 100 001 ; 8 573 380 ;
854 732 457 ; 213 874 406 900 ;
8 000 005 700 .

4- In the number 7 777 what is the place value of each 7 digit?

5- In the number 12 675 425,
what is the number of:
Millions? Thousands? Hundreds? tens?

6- Follow the example to complete below:
5 743 = (5 × 1000) + (743 × 1)
3 645 128 = (..... × 1000 000) + (645 × .....) +
                (128 × .....)
645 385 000 = (645 × 1 000 000) + (..... × 1000)
381 754 000 = (......) + (......).
12 958 000 = (......) + (......).
0 437 000 = (......) + (......).
30 885 438 = (......) + (......).
500 600 900 = (......) + (......) + (......).
3 423 745 609 = (......) + (......) + (......) + (......).

7- Decompose the following numbers:
7 840 = (7×1 000)+(8×100)+(4×10)
89 635 = ; 283 540 = ; 2 647 850 = ; 15 738 200 =

8- Find the number corresponding to the following decompositions:
(4×1000)+(7×100)+(8×10) =
(5×1 000 000)+(8×1000)+(9×100) =
(6×1000)+(4×100)+(5×10) =
(3×1 000 000)+(7×1000)+(5×100) =
(5×1 000 000 000)+ (8×1 000 000)+(7×1000)

9- Write in the letters, then decompose each of the following numbers:
7 987 ; 61 004 ; 100 001 ; 3 000 029 ;
64 300 521 ; 2 368 542 378 ; 1 245 ;
745 288 693 542 .

10- For the number 18 315 407,
indicate the digit of: hundred of thousands; tens of thousands; millions;
What will you obtain if you add to it four hundred thousands?

11- Find the number with 7 in the ones digit that includes 8 millions, one hundred twenty-five units of thousands and fifty-four tens.

12- What number do you obtain
if we subtract 4 tens of thousands from 957 832?
if we add 4 hundreds of thousands to the obtained number?
If we add 5 tens of thousands to the preceding results?

13- Write in digits each of the following numbers:
67 hundreds of thousands.
5 millions 8 hundreds.
10 hundreds of thousands.
1000 millions.

14- Rewrite the numbers given below by decreasing order then write them in letters.

375 642 ; 851 915 ; 87 635 401 ;
236 418 000 ; 5 387 465 421 ; 100 009 .

1- Is it true or false that:
- Four million five hundred thousand three hundred corresponds to 500 300?
- Six hundred forty-two million four corresponds to 60 042 004?

2- Write in letters then decompose each of the following numbers:

128 754;   13 723 415;   5 315 675 401.

Problems

1- How many times do you pronounce the word:
- "nine" when you count till one hundred?
- "twenty" when you count till one hundred?
- "hundred" when you count till one thousand?

2- In Beirut, 400 000 persons use the elevator everyday.
- How many persons use it during a week? A month? A year?.
- Write in letters, then decompose each answer.

3- Astronauts use the astronomic unit (*AU*) that is equivalent to 150 000 000 *km*. Astar is 245 000 *AU* far from Earth.
- Calculate this distance in:- thousands of kilometers;- millions of kilometers.

4- Out of the six following numbers:
312 ; 238 ; 125 ; 124 ; 326 ; 619
four have as a sum 1 000. What are they?

5- How many thousands are there in:
- a million? ; - a billion ?

6- Find all the numbers included between 500 000 and one million that can be written with three 7 digits and three 2 digits.

7- In 1996, the Arab countries produced the following quantities (in thousands of tons) of Petroleum:

Saudi Arabia	428 000
Bahrain	5 200
United Emirates	117 300
Iraq	28 800
Kowait	107 200
Qatar	21 800
Syria	31 400
Oman	44 300
Yemen	17 500

- Calculate the total production in: thousands of tons; in tons; in kilograms.

8- To the number 759 843,
- Successively add a ten and two thousands, a ten and five hundreds.
- Verify your answer using your calculator.

9- I have 13 bills of hundred thousands of LL; 10 bills of 500 thousand LL; 8 bills of 20 thousand LL; 12 bills of 10 thousand LL; 18 bill of one thousand LL.
- What is my capital? write it in letters.

Just for fun

This calculator has 4 keys:

On   Off   +10   ×10

When you push "ON",
the calculator displays 1; and when you push, it immediately displays the result of the operation.
- How many times at a minimum should you press these keys for the calculator to display 100? 1 000? 1 000 000?

Lines and circles

3

Objectives

At the end of this chapter, I will be able to:

- Determine the position of a straight line with respect to a circle. Knowing the average distance from the center of the circle to the straight line.

Activities
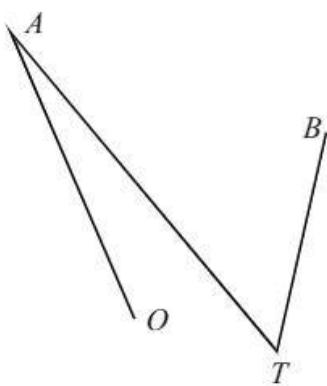
a) Reproduce the opposite figure then draw using different colors each of the lines (OA), (AT), (BT) and (AB).

b) Without doing any other drawings, determine the lengths in cm of the distances from point O to each of the lines (OA), (AT), (BT) and (AB).

c) Construct the circle ($\mathscr{C}$) of center O and radius OT; then color one of the regions of the disc limited by circle ($\mathscr{C}$) and straight line (BT).

d) Reproduce and complete the table:

Line	(AO)	(AT)	(BT)	(AB)
Number of common points with circle ($\mathscr{C}$).	—	—	—	—

e) Does the distance from the center O of a circle ($\mathscr{C}$) to a straight line (D) allow to determine the number of common points ($\mathscr{C}$) and (D)? How?

21

(D)

(𝒞)

O R

d

$d > R$
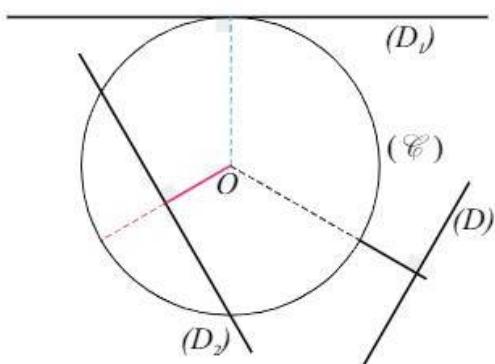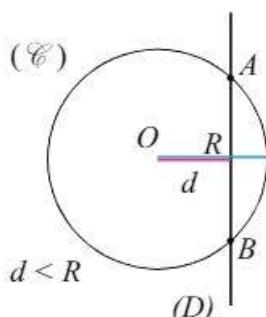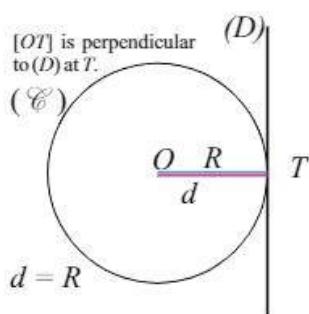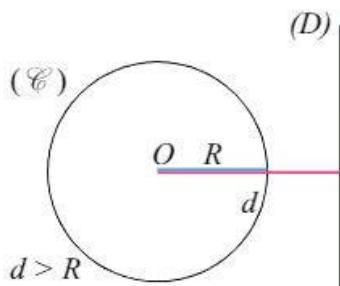
A straight line (D) can take different positions from a circle (𝒞) in a plane.

● The line (D) is external to (𝒞)

We say that (D) is external to (𝒞) when (D) has no points in common with (𝒞).

This is the case when the distance d from the center O of circle (𝒞) to the line (D) is greater than R, the radius of this circle.

[OT] is perpendicular to (D) at T.

(D)

(𝒞)

O R *T*

d

$d = R$

● The line (D) is tangent to (𝒞)

When (D) and (𝒞) have one point in common we say they are tangent The only point in common is called tangency point or contact point.

When (D) is tangent to (𝒞), the distance from O center of (𝒞) to (D) is equal to R, radius of circle (𝒞).

(𝒞)

A

O R

d

B

$d < R$

(D)

● The line (D) is secant to (𝒞)

If the line (D) cuts the circle (𝒞) in two distinct points A an B, then (D) is secant to (𝒞). In this case, the distance from O center of (𝒞) to (D) is less than R, radius of circle (𝒞),

and the circle (𝒞) is cut into two parts called "arcs".

The segment [AB], part of (D) internal to circle(𝒞), is called "chord".

The longest chord in a circle is the diameter. The arcs determined by the diameter are the two semi-circles.
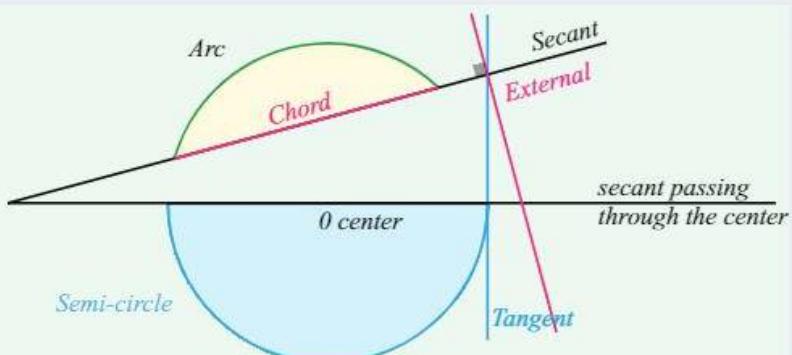
Exercises

1- In how many ways can you complete the figure to obtain:

a) a rectangle?

b) a parallelogram?
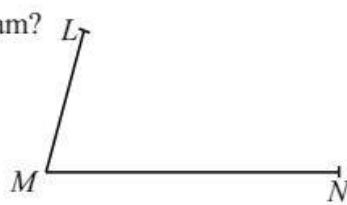
c) any quadrilateral?

2- Answer by "true" or "false":

Starting from one point taken out of a straight line (D), we can draw:

• one perpendicular to (D);
• many perpendiculars to (D);
• only one parallel to (D);
• 4 parallels to (D);
• one perpendicular and one parallel to (D).

3- Reproduce on your notebook and complete by "parallel" or "perpendicular":

• In a rectangle,
The opposite sides are.................
The adjacent sides are ....

• In a square,
The adjacent sides as well as the diagonals are ....; the opposite sides are ....

• In a rhombus,
The diagonals are ....
The opposite sides are................... .

Properties of parallel and perpendicular straight lines

4- Two distinct lines (D) and (D') have point A in common. Can they have other points in common? How will they be in this case?

5- Draw two parallel straight lines (AB) and (CD) then a secant (d) that cuts (AB) in A. What can you say about (d) and (CD)?

6- ABC is a triangle. M is the mid-point of side $[BC]$.
Draw the respective perpendicular $[BN]$ and $[CP]$ to (AM).
Compare the distances of B and C to (AM).

Self-evaluation

The point O is situated at $2cm$ from (d), at $3\,cm$ from (D) and at $4\,cm$ from (D')
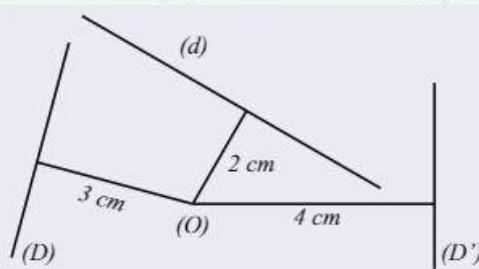
Reproduce and complete the table with the number of common points to the circle and to the lines $(d), D)$ and (D'):

	Circle of center O and radius			
	1.8 cm	2 cm	3 cm	4 cm
(d)				
(D)				
(D')				

1- Given a triangle *ABC*. Draw the heights (*AA'*), (*BB'*) and (*CC'*) of *ABC*. What can you say about them?

Draw from *A* the perpendicular (*MP*) to (*AA'*).

Draw from *B* the perpendicular (*MN*) to (*BB'*).

Draw from *C* the perpendicular (*NP*) to (*CC'*).

What can you say about segments [*MB*] and [*BN*], [*AM*] and [*AP*], [*NC*] and [*CP*]?

2- Given two parallel lines (*D*) and (*D'*).

• Locate the points *A*, *B*, *C*, *D* and *E* equidistant from (*D*) and (*D'*).

• Draw the circles of center *A* and of respective radius: 2 *cm*, 3 *cm* and 6 *cm*.
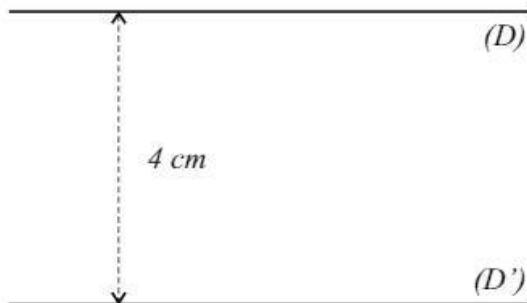

3- Draw the tangent (*D*) at point *T* to circle (*C*) cut (*OB*) at *A*. Determine *T'*, the symmetric of *T* with respect to (*OB*).

Does *T'* belong to the circle? Justify.

Draw the tangent (*D'*) at *T'* to the circle (*C*).

What can you say about (*D*), (*D'*) and (*OB*)?
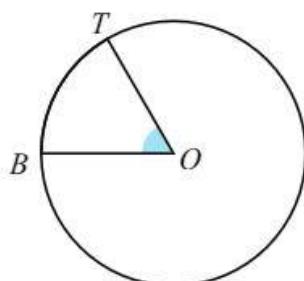
4- *AA'* is the diameter of a circle (*C*) of center *O*.

• Draw the tangents (*Ax*) and (*A'y*) to (*C*).

• Are (*Ax*) and (*A'y*) parallel? Why?

5- Calculate the radius of a circle of center *O* admitting, as tangents:

a) (*AB*)

b) (*AD*)

c) (*BC*)
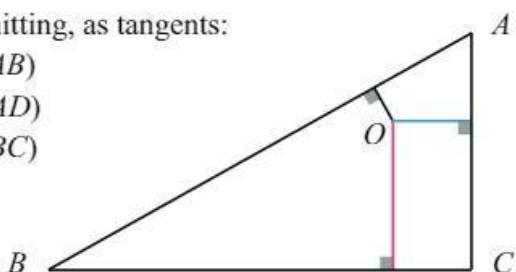
6- What is the measure of the radius of the circle of center *O* that has:

a) (*AC*) as tangent?

b) (*BC*) as secant?

7- Draw a parallelogram *ABCD*, then the circle (*C*) of diameter [*CD*].

• (*C*) cuts (*AD*) at *M* and (*DB*) at *N*. Measure using a protractor each of angles $D\widehat{M}C$ and $D\widehat{N}C$.

• P being the point of intersection of (*C*) with (*AC*), calculate the value of angle $D\widehat{P}C$ in degrees.
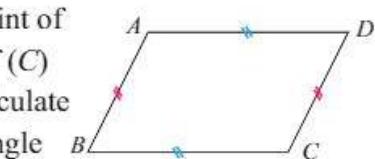
8-

• Draw the circle of diameter [*OA*]. It cuts the circle of center *O* in *B* and *C*.

Use the protractor (or set square) to find the value of angle $O\widehat{B}A$ and $O\widehat{C}A$ in degrees.

• What do (*AB*) and (*AC*) represent to the circle of center *O*?

9-
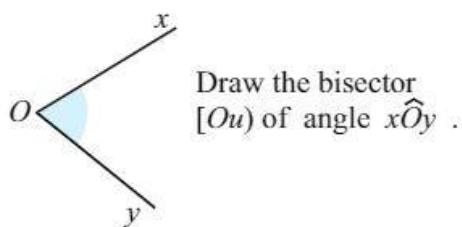
Draw the bisector [*Ou*) of angle *x*Ô*y* .

- Choose a point *A*, distinct from *O*, on the semi-line [*Ou*).
  Determine the distances from *A* to each of the semi-lines [*Ox*) and [*Oy*).
  Draw a circle of center *A* tangent to [*Ax*) and [*Oy*).
- Redo the same with two other points *A* and *C* of [*Ou*).

10- Given two parallels (*D*) and (*D'*):
Draw a circle, in each case, that has:

(*D*)

2,5 cm

(*D'*)
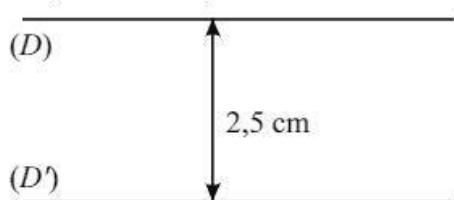
a) (*D*) and (*D'*) as tangents;
b) (*D*) as tangent and (*D'*) as secant;
c) (*D*) and (*D'*) as secants;
d) (*D*) and (*D'*) as two external lines.

**11-** Draw a circle of center *O* and radius of 4 cm.

- From a point *A* exterior to the circle, draw a secant that cuts the circle at *M* and *N*.
- Determine the points *P* and *Q* symmetry to *M* and *N* respectively relative to the straight line (*AO*).
- What can you say about lines (*MQ*) and (*NP*)? Justify your answer.

12- Draw a circle (*C*) of center *O* that has radius of 3 *cm*. Let [*AB*] be a diameter in this circle.
Using the compass, draw the circles of centers *A* and *B* and of radius *AO* and *BO* respectively. These two circles cut (*C*) in *M*, *N*, *P* and *Q* respectively.

Compare:
a) The lengths of chords [*AM*], [*MP*], [*PB*], [*BQ*], [*QN*] and [*NA*];
b) The measures of angles *A*Ô*M*, *M*Ô*P* *P*Ô*B*, *B*Ô*Q*, *Q*Ô*N* and *N*Ô*A* .

- Calculate lengths of arcs: $\overset{\frown}{AM}$, $\overset{\frown}{MP}$ and $\overset{\frown}{PB}$ of this circle.
- Find a way to cut the circle into 6 equal arcs.
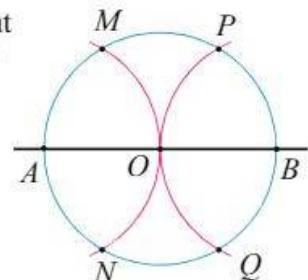
13- Determine the length of the arc colored in red on the circle of radius 50 *mm*.
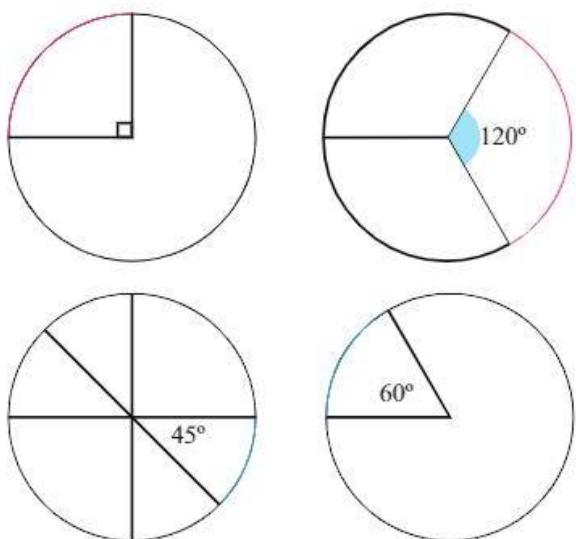
15- Compare the lengths of chords [*MN*], [*AB*] and [*DC*].
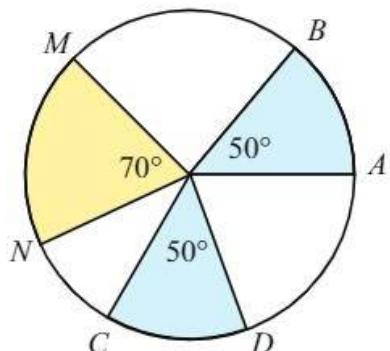
16-
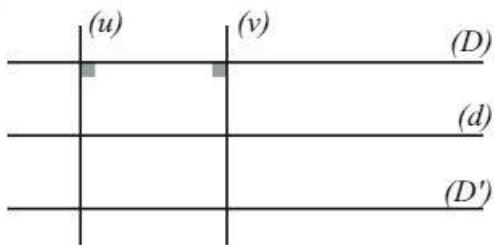
Given:

(D) parallel to (D')

(d) parallel to (D)

(u) and (v) are perpendicular to (D).

What can you say about

(d) and (D')?   (u) and (v)?   (v) and (d)?

17- Given:

"Locate three points A, B and C and draw from each a parallel to a straight line (D)".

Here are the answers of 3 students:

Sami: "I obtained 3 distinct parallel lines to (D)."

Samar: "I could only make two parallels to (D)!"

Samer: "I could not make but only parallel to (D)!".

Knowing that all 3 students are correct, determine the position of points A, B and C used to draw the parallels to (D).

Reproduce this drawing by copying once only one of these two figures of which it is formed: The circle, or the square, or one of the parallelograms, or one of the triangles..
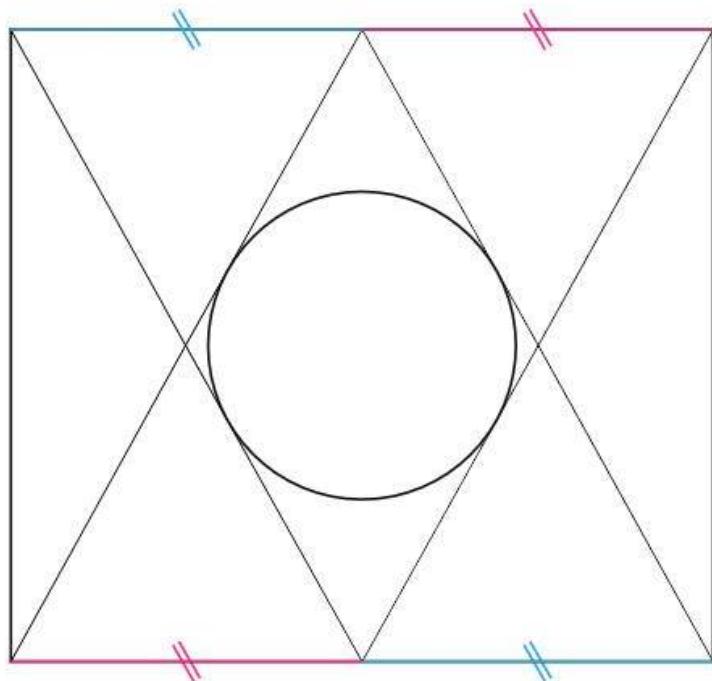
L.C.M. and G.C.D. of two whole numbers

4

Objectives

At the end of this chapter, I will be able to:

- calculate the L.C.M and G.C.D of two whole numbers.
- recognize 2 numbers prime with each other.

Activities

a) A physician visits a clinic on all days that are multiples of 2. The dentist, however, visits the same clinic on days that are multiples of 3. If the first visit of the doctor on the 2nd of November and the first visit of the dentist on the 3rd of November:

Circle the dates corresponding to the doctor's visits.

Square the dates corresponding to the dentist's visits.

Write a list of the dates when both doctors meet in the clinic.

What is the L.C.M of 2 and 3?

What is the L.C.M of 7 and 14?

November 1998

S	1	8	15	22	29
M	2	9	16	23	30
T	3	10	17	24	
W	4	11	18	25	
Th	5	12	19	26	
F	6	13	20	27	
Sa	7	14	21	28	

b)

Number	Divisors	Communs Diviseurs of:
14		14 and 15 :
15		14 and 42 :
42		15 and 42 :

- What is the greatest common divisor of 14 and 42? of 14 and 15? of 15 and 42?

- What is the common divisor of 14 and 15?

3 is a divisor of 12
4 is a divisor of 12
We also say
3 divides 12
4 divides 12
Are 16 and 25 prime with each other? Why?

The multiples of 5 are:
  0, 5, 10, 15, 20, 25,......
The multiples of 4 are:
  0, 4, 8, 12, 16, 20, 24,...
Common multiples are:
  0, 20, ..., ...
L.C.M of 4 and 5 is
  $\boxed{20}$

L.C.M. of 6 and 12 is
  $\boxed{12}$
The divisors of 18 are:
1, 2, 3, 6, 9, 18.
The divisors of 24 are:
1, 2, 3, 4, 6, 8, 12, 24.
The common divisors are:
1, 2, 3, 6.
The G.C.D of 18 and 24 is
  $\boxed{6}$

Divisors of 14 are: 1, 2, 7, 14.
Divisors of 17 are: 1, 17.
G.C.D (14,17) = $\boxed{1}$.

a) In order to find the lowest common multiples of two integers:

- we write the list of all multiples of the first integer then of the second one.

- pick up then the common multiples of both lists.

- the smaller one (different from zero) is called the lowest common multiple of these two numbers.
 If one of the two numbers is a multiple of the other, then the greater one will be their L.C.M.

b) In order to find the greatest common divisor of two integers:

- we write the list of divisors of each.

- we pick up the common divisors of both lists.

- the greatest one will be called the greatest common divisor of the two integers.
  If the integer m is a divisor of n then m is the G.C.D of m and n.
 Two integers are prime with each other if their G.C.D is 1.

Focus

The L.C.M. of 12 and 18	The G.C.D of 12 and 18
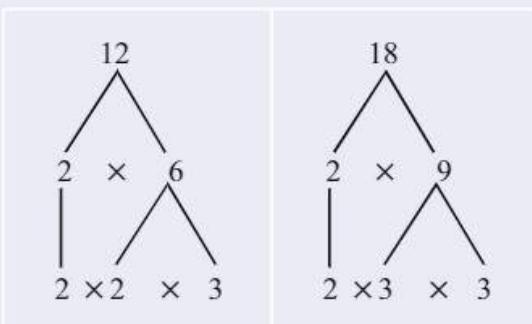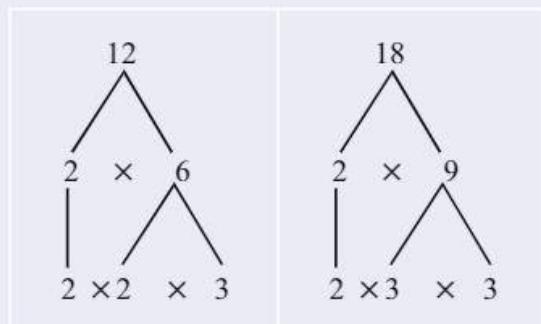
L.C.M (12,18) = (2 × 2) × (3 × 3) = 36   G.C.D (12,18) = 2 × 3 = 6.

Exercises

1- Write the list of all multiples, less than 100 of each of the following: 3, 5, 8, 11, 18, 45.

2- Find the L.C.M. (other than zero) of each of the following pairs:

9 and 18	12 and 24	13 and 26
4 and 6	8 and 10	9 and 12
14 and 21	20 and 30	24 and 36.

3- Using your calculator:

- Find and write the first twenty multiples of 16, then the first twenty multiples of 40.
- Find the L.C.M of 16 and 40.

4-

- Find the greatest multiple of 7 which is inferior to 200.
- What will be the next multiple?
- Same question for the multiple of 21 that is less than 300.

5- Write the list of divisors of 15; 0f 18; of 24; of 30; of 48.

- Write the list of the common divisions to 18 and 24.
- Compare this last one with the list of divisors of 6.

6- Find the G.C.D of each of the following pairs:
7 and 14; 17 and 34; 37 and 74; 15 and 25; 68 and 102; 72 and 96; 300 and 700; 6000 and 12000.

7- Decompose into prime factors the numbers 18 and 24 (using the tree-method).

- Find the L.C.M and the G.C.D of 18 and 24.
- Calculate the product of L.C.M and G.C.D
- Find the product of 18 by 24. What do you notice?

8- The G.C.D of 180 and a number m (which is less than 180) is 15.

- Find m.

9- Find the G.C.D of the following pairs: 13 and 14; 24 and 29; 63 and 64; m and $m+1$.

Self-evaluation

1- Find L.C.M and G.C.D of each of the following pairs:

34 and 68; 16 and 24; 180 and 270; 45 and 90 .

2- Choose from the following given pairs those that are prime with each other:

9 and 16; 36 and 37; 48 and 51; 180 and 181 .

Problems

1- Who am I?
- I am a common multiple of 5 and 12 and I am less than 65.
- I am a divisor of 24, odd, and different from 1.
- I am the G.C.D of 35 and 42.

2- I think of a number which is a multiple of 8. If I subtract 1 from it, I obtain a multiple of 5 and 11 at the same time.
- Find this number, knowing that it is greater than 30 but less than 60.

3- Find the G.C.D and L.C.M of 30 and 48.
Verify that G.C.D × L.G.M is equal to 30 × 48.

4- The G.C.D of 32 and 48 is 16.
- Find in two different ways the L.C.M of 32 and 48.

5- Find all pairs of prime numbers between 20 and 30.

6- A lighthouse emits 2 signals: red and green.
The red signal is emitted every 45 seconds, and the green every 30 seconds.
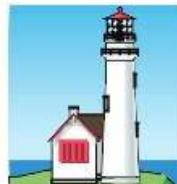
The two signals are emitted together at 10 hours.
- How many times will they be emitted together in 30 seconds?
  24 hrs?
  12 days?

7- Find the smallest integer admitting 6 divisors ; 8 ; 12 .

8- Find two integers knowing that their G.C.D is 2 their L.C.M is 36.

9- Find two prime numbers if their product is 72 (many solutions).

10- If we arrange the books in a bookcase by placing 12 per row or 16 per row, 4 books remain non-arranged. Knowing that the number of books is between 250 and 300, find this number.

11- Find two prime numbers whose sum is 26 and L.C.M 105.

12- Using your calculator, find the L.C.M and G.C.D Of each of the following pairs of numbers: 19 and 20; 48 and 72; 360 and 540.

Just for fun

Reem picks up flowers that are equal in numbers to her age.

On each day that follows, she picks up twice as many flowers as she picked up on the day that preceded. On the fourth day, she found out that she could make a bouquet where the number of flowers is equal to the number of years of her grand mother's age. Knowing that her grand mother is 60 years older than her, find Reem's age.

Patrons and solids

**5**

Objectives

At the end of this chapter, I will be able to:
- construct a solid of known patron.

Activities

Activity 1

Here are different representations of same solid (*S*) where their faces have different colors.
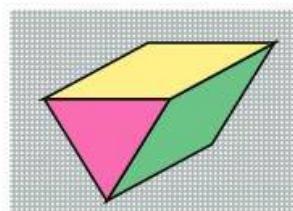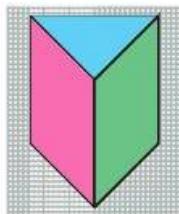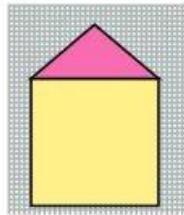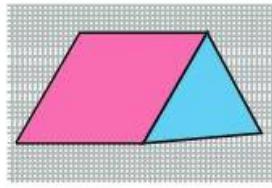
a) How many vertices (*S*) has? Sides? Faces? Square faces? Triangular faces?

Determine the colors and the form of the opposite faces of solid (*S*). Are these faces similar?

b) Which of the drawings below represent the faces of (*S*)?
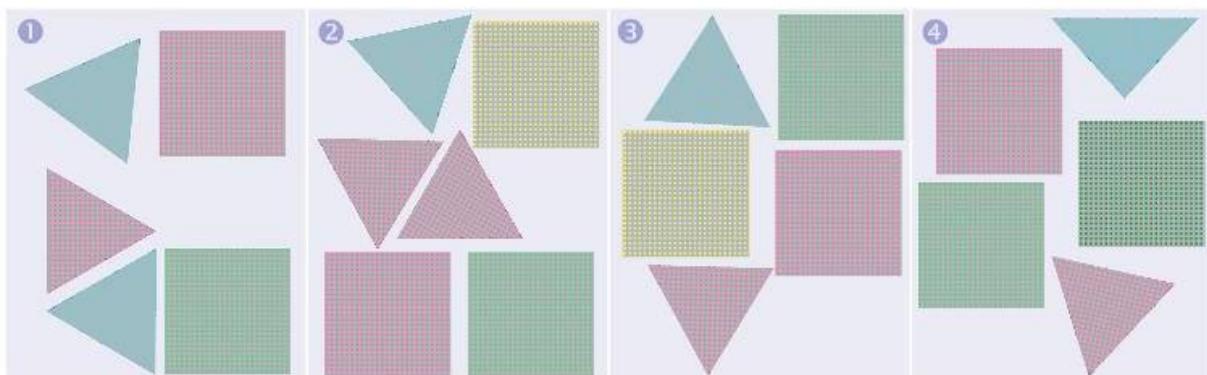
c) One of the five figures below is a patron of (*S*). Choose this patron and justify your choice.


a)

b)

c)

d)

e)
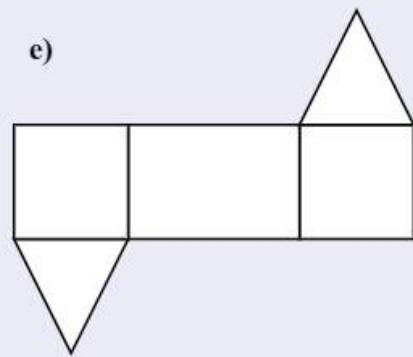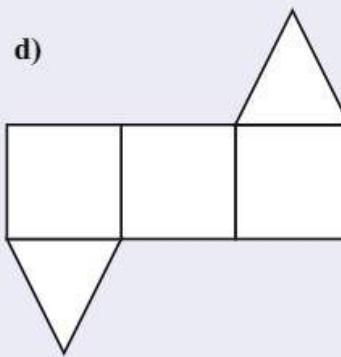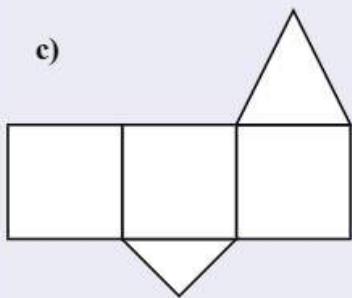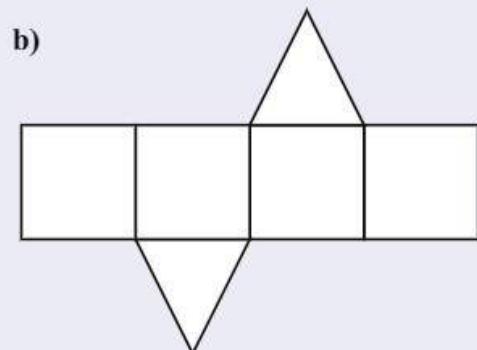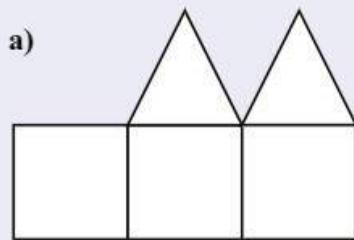
Explain why each of the other four figures cannot be a patron of the solid (*S*).

d) Construct on a paper two distinct patrons of (*S*) and mark on each of them the sides that will coincide upon folding.

e) What procedure would you follow to draw the patron of (*S*)?

Activity 2

The balloon and watermelon showing below resemble each other in such a way that you cannot distinguish between them.

a) From the list of words: ball - empty ball - ball-skin - sphere , pick up those characterizing:

1) The watermelon	2) The balloon

b) By cutting the balloon and the watermelon we obtained the figures given next:

Besides the color, how can you recognize the parts of the watermelon? The parts of the balloon?
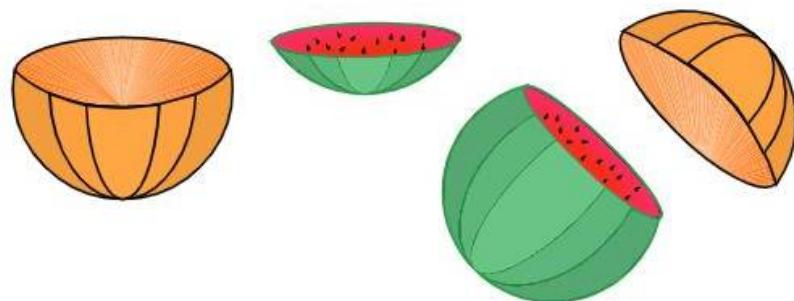
c) What is the print on the plane of the table of:
a semi-balloon?   A semi watermelon?
Draw these figures in your notebook.
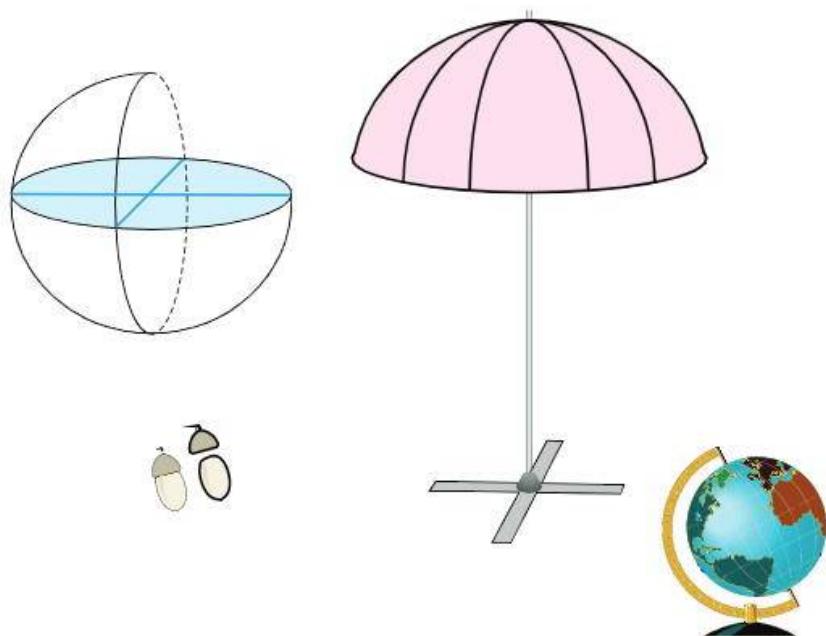
The construction of a patron of a solid (S) consists of:

(S)

- Knowing the number of vertices, sides and faces of this solid.

5 vertices
8 sides
5 faces
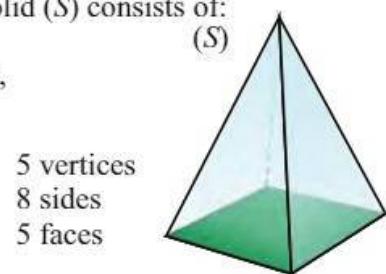
- Recognizing the form of faces of (S) with their dimensions in order to reproduce them.

- Putting the reproduced forms in a plane figure, respectively the "adjacent" and "opposition". From these figures we can obtain solid (S).

(P)
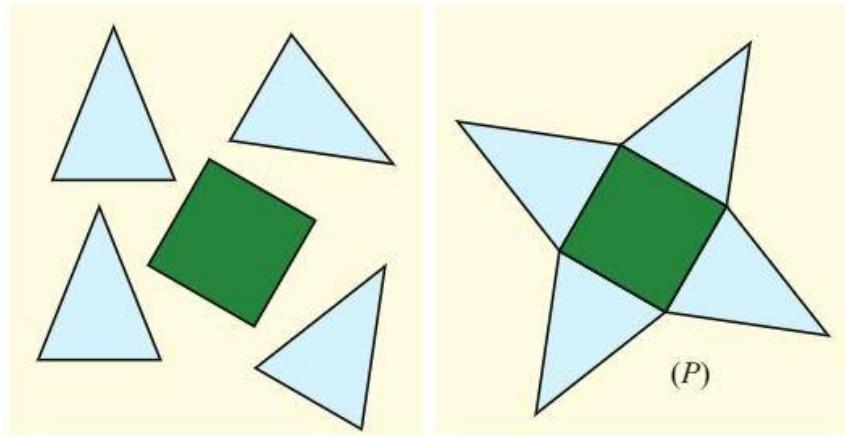
Form a patron to a solid and vice-versa:
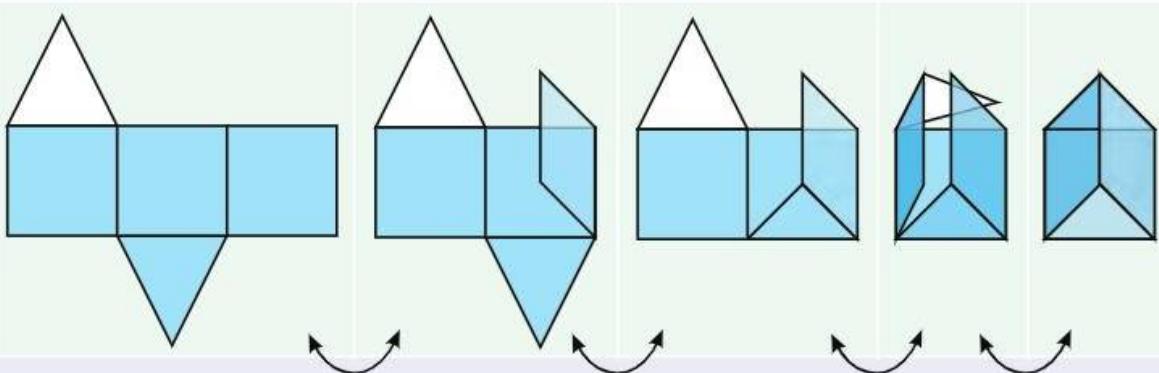
**1-** Find the solid that corresponds to each of the following patrons:

a)

b)

c)

d)

e)
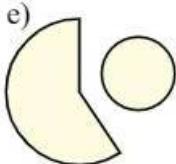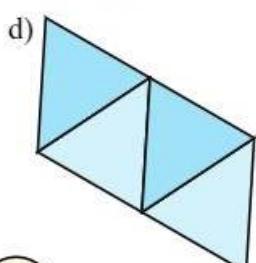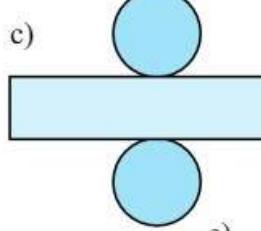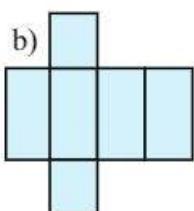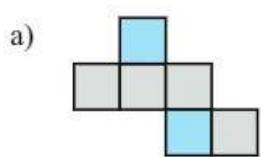
2- Correspond each solid to its patron

a)

b)

c)

d)

e)

f)
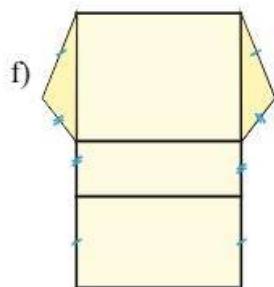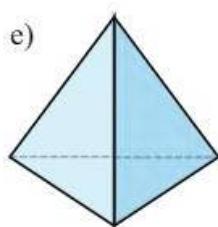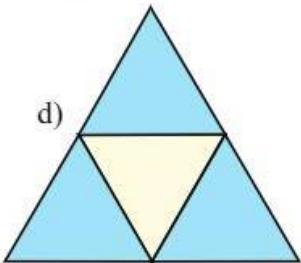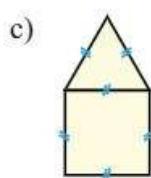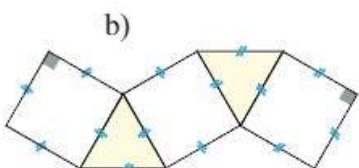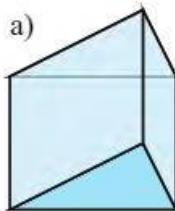
3- Construct the solid that corresponds to each of the following patrons:
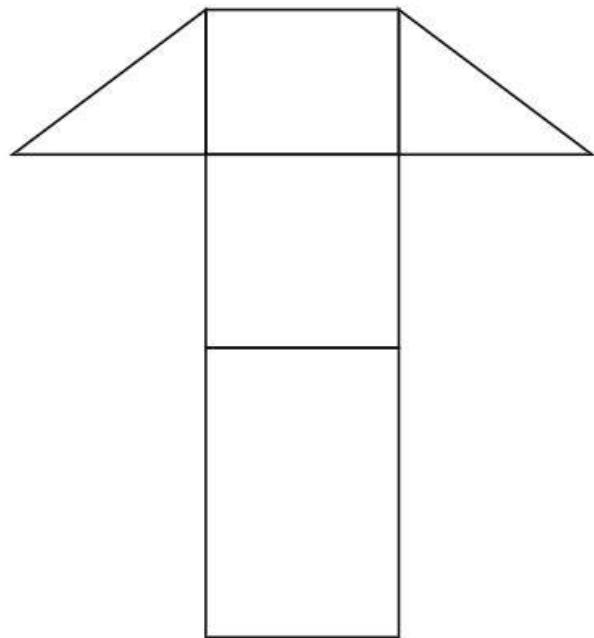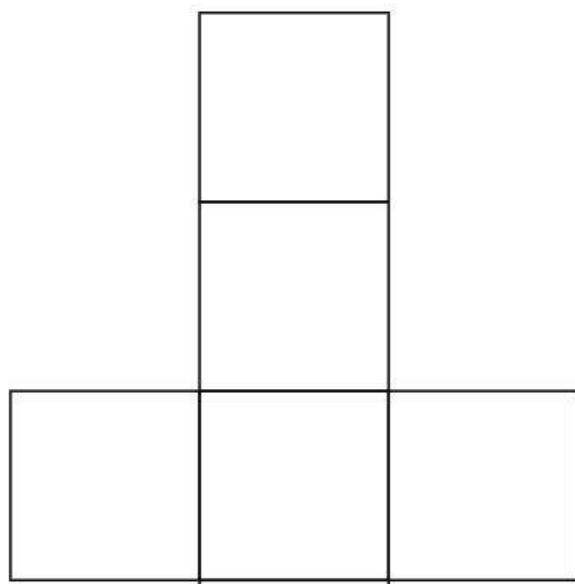
Which of the following patrons represents a cube?
Why are not the others ?
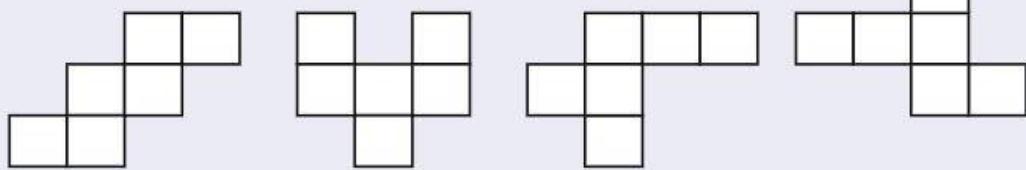
Problems

1- a) Calculate the total lengths of the sides of the solid given below:

2 cm

3 cm

4 cm
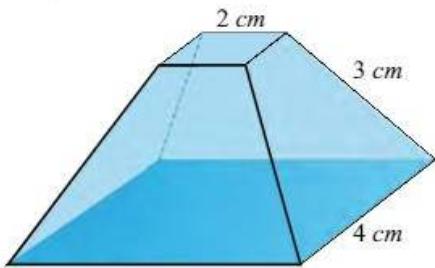
b) Draw a patron for this solid, then calculate its perimeter in 2 different ways.

c) Compare this perimeter to the total length calculated in a).

2- The rubber represented below is formed of three glued solids.

4 cm

2 cm

1.5 cm

2.5 cm
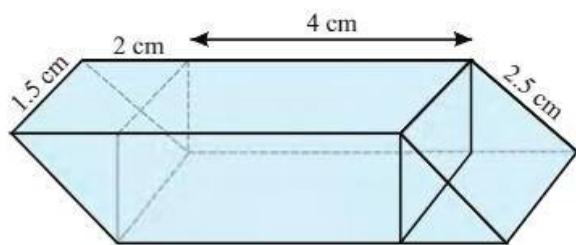
a) Draw a patron for each of three solids.

b) Imagine then draw a sac for this rubber.

Just for fun

Starting from a cube patron and a pyramid patron of square base, draw a patron for this given solid.
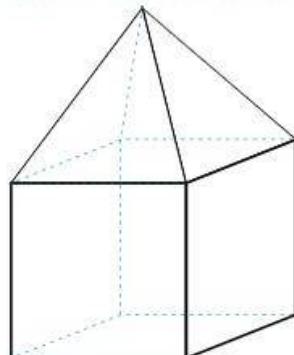
Powers

Objectives

At the end of this chapter, I will be able to:

- calculate powers of exponent 2 or 3.
- calculate powers of 10.
- decompose an integer using powers of 10.

Activities

Activity 1
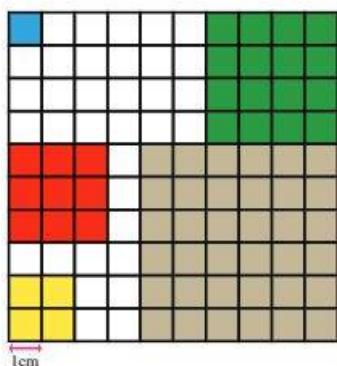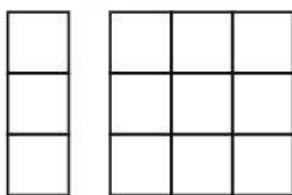
I- a) Remember the formula that determines the area of a square of side S: $\mathcal{A} = \ldots\ldots\ldots$

- Complete the following table:

Square	Area of the square (in cm^2)		
Blue	1×1	1	1^2
Yellow	$2 \times \ldots$	4	
Red			3^2
Green			
Brown			
big square	$10 \times \ldots$		

- Complete the following equalities:

$5 \times 5 = \ldots\ldots\ldots = 5^2$ $9 \times 9 = \ldots\ldots\ldots = \ldots\ldots\ldots$

$7 \times 7 = \ldots\ldots\ldots = 7^{\cdots}$ $11 \times 11 = \ldots\ldots = \ldots\ldots\ldots$

$8 \times 8 = \ldots\ldots\ldots = \ldots\ldots$ $12 \times 12 = \ldots\ldots = \ldots\ldots\ldots$

b) Complete the following tables:

$25 \times 25 = 25^2$

	Product of a number by itself	Simplified writing	Product
1	1×1	1^2	1
2	2×2		
3	3×3		
4			16
5		5^2	

	Product of a number by itself	Simplified writing	Product
6	6×6		
7		7^2	
8	8×8		
9		9^2	
10			100

1cm

c) Use your calculator to find the following products:

$$325 \times 325 \qquad ; \qquad 325^2 \qquad ; \qquad 325 \times 2$$

What do you notice?
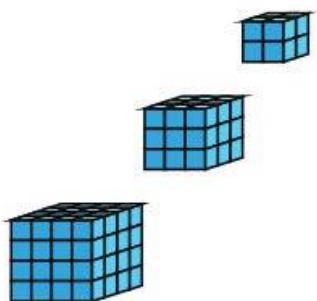
II-a) With how many small cubes ⬜ can we fill the large cube given next?
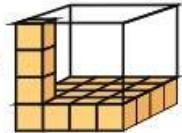
b) Complete: $4 \times 4 \times 4 =$ ....

Simplify the writing $4 \times 4 \times 4$ by writing $4^{\cdots}$

c) With how many cubes of side $1 cm$ do we need to fill a cube of side $2\ cm$?

Give the result in a simplified form ⬜ $^{\cdots}$

Same question for cubes of sides equal to $3\ cm$; $5\ cm$; $6\ cm$; $7\ cm$; $8\ cm$; $9\ cm$; $10\ cm$.

d) Complete the following table:

Product of 3 equal factors	$1 \times 1 \times 1$	$2 \times 2 \times 2$				$6 \times 6 \times 6$	$7 \times 7 \times 7$	$8 \times 8 \times 8$		
Simplified Writing	1^3	2^3	3^3						9^3	
Product	1	8		64	125					1000

e) Using your calculator, find:

$$15^3 \qquad ; \qquad 15 \times 15 \times 15 \qquad ; \qquad 15 \times 3$$

What do you notice?

Activity 2

$20^3 = 20 \times 20 \times 20$

a) How many times should we multiply 10 by itself to obtain 1000? One million? One billion?

Avoid the repetitive writing of the product in the following equalities:

$1\ 000\ 000 = 10^{\cdots}$
$1\ 000\ 000\ 000 = 10^{\cdots}$

b) Write the decomposition of the number 24 057 by the powers of ten with their simplified writing:

$$(2 \times 10\ 000) + (4 \times 1\ 000) + (0 \times 100) + (5 \times 10) + (7 \times 1).$$

c) Write the number appearing in the following sentences as a product of an integer by powers of 10:

"The distance from the Sun to planet Mars is approximately 228 100 000 km".

$1\ 000\ 000\ 000\ 000 = 10^{12}$

d) Using your calculator find:

$315.27 \times 10 =$.......... $315.27 \times 10^3 =$ ....

$315.27 \times 10^2 =$ .... $315.27 \times 10^4 =$ ....

What do you notice?

$$7 + 7 = 2 \times 7$$
$$7 + 7 + 7 = 3 \times 7$$

$$7 \times 7 = 7^2$$
$$7 \times 7 \times 7 = 7^3$$

We use multiplication to simplify the repetitive sum writing. Similarly, we use powers to simplify the product-writing of many equal factors.

- The product of two equal factors of 7 is written as 7^2.
- The product of three equal factors of 7 is written as 7^3.

Who am I?

I am the number 63 and
$$63 = 7 \times 9$$
$$= 7 \times 3^2$$

- I am the square of 7 and read as "7 squared"
- I simplify the writing of 7×7
- I am a power of 7
- I am the writing of 49 as a power
- my exponent is 2
- I am equal to 49
- I am the base
- I am a power of 7
- I am the exponent
- I am the cube of 7 and read as "7 cubed"

$$7^2$$

$$7^3$$

I am the Number 73 my only writing is 73^1.

$$5\,000\,000 = 5 \times 10^6$$
$$1\,000\,000\,000 = 10^9$$

The number of zeros is equal to the exponent.

Who are we?

We are the powers of 10.
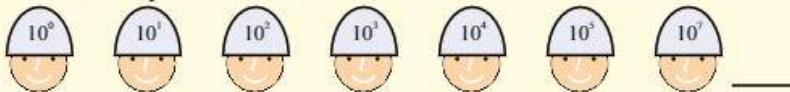
- We are used to:
  - decomposing numbers:
    $3\,478 = (3 \times 10^3) + (4 \times 10^2) + (7 \times 10^1) + (8 \times 1)$.
  - simplifying the writing of numbers ending with zeros:
    $5\,000\,000 = 5 \times 10^6$
- When we are multiplied by a decimal number, we move its decimal point to the right, as many places as the exponent of power of 10: $4.265 \times 10^2 = 426.5$; $4.655 \times 10^4 = 46\,550$
- We add a zero to have as many places after the decimal point as the exponent of power of 10.

The product of many factors equal to a number "a" which is called power of a.

5^2 is read "five to the power two" or "5 squared".

8^3 is read "8 to the power 3" or "8 cubed".

10^3 is a power of 10.

- $10^0 = 1$
- $10^1 = 10$
- $10^2 = 100$
- $10^3 = 1\,000$

- We can use powers of 10: $10^0; 10^1; 10^2; 10^3; ....$
  - to decompose an integer: $4\,385 = (4 \times 10^3) + (3 \times 10^2) + (8 \times 10) + (5 \times 1);$
  - to simplify the writing of a number ending with zeros:

$14\,000\,000\,000 = 14 \times 10^9$ *(exponent indicates the number of zeros)*

  - to calculate the product of one of them by a decimal number:

$$43.126 \times 10^2 = 4\,312.6 \qquad\qquad 8.2\,650 \times 10^4 = 82\,650.$$

We move the decimal point to the right as many places as the exponent of 10.

Exercises

1- Circle the powers having 2 for exponent:

2^3 ; 2^5 ; 2^2 ; 10^2 .

2- Underline the power of base 3:

3^2 ; 10^3 ; 5^2 ; 4^3 ; 3^3 .

3- Choose the right question:

$25^2 =$ 25   50   625

4- Calculate:

$24^2 = ...$ $10^4 = ...$
$30^3 = ...$ $10^7 = ...$
$13^2 = ...$ $8^3 = ...$

5- Write each of the following numbers as a power of 10:

1 million $= 10.....$
1 billion $= ....$
$100\,000\,000 = ...$

6- Write each of the following numbers as a product of an integer by the power of 10:

$38\,900\,000$ $= ....$
$510\,000\,000$ $= ....$
$4\,592\,000\,000$ $= ....$
$6\,000\,000\,000$ $= ....$

7- Use your calculator to find:

a) $2\,139^2$; $2\,139 \times 2\,139$; $2\,139 \times 2$.
b) 281^3 ; $281 \times 281 \times 281$; 281×3.

What do you notice?

8- Using your calculator, find:

3.75×10^3 $=$
0.02×10^4 $=$
85.005×10^5 $=$
37.015×10^2 $=$
$0.12\,504 \times 10^6$ $=$

9- Calculate:

$2^3 + 3^2 =$

$1 + 10 + 10^2 + 10^3 + 10^4 =$

$5^2 \times 4^3 + 10^2 \times 2^3 - 10^3 =$

10- Using your calculator, find:

$328^2 \times 10^3 - 25^3 \times 10^2 =$

$1423^2 \times 10^2 - 38^3 \times 10 =$

$5 \times 10^4 - 13 \times 10^3 =$

11- Decompose the following numbers using powers of ten:

3 281 - 35 004 - 721 059 - 6 780 507 - 90 000 801

12- Write the integer corresponding to each of the following operations:

a) $24 \times 10^4 - 2.5 \times 10^3 - 0.03 \times 10^5$

b) $(5 \times 10^7) + (8 \times 10^6) + (3 \times 10^3) + (2 \times 10^2) + (9 \times 1)$

Self-evaluation

1- Calculate: $23.80\,013 \times 10^7 =$; $0{,}0\,002\,435 \times 10^9 =$

2- Correct the mistakes appearing in the following sentences:

a) 9 is the exponent of the power 9^2 ; b) 3 is the base of the power 4^3.

c) 6^2 is read as "2 exponent 6".   ;   d) One million is a power of 10 in which the exponent is 5.

e) 2^5 is a power of 5.

Problems

1- Write the numbers that figure in the sentences below as a product of an integer by powers of 10:

- The celerity (speed) of light is 300 000 *km* per second.

- Lebanon needs 3 300 millions of m^3 of water per year. The average volume of water from precipitation reaches 8600 millions of m^3 per year.

- The distance between the Earth and the Sun is 150 million *km*.

- In the year 2 025, the world population will reach 9.3 billions.

- In the year 2 010, Lebanon will need 2 160 000 000 m^3 of water for irrigation.

2- During a race, Ghassan ran a distance equivalent in meters to the cube of 18, and Nada a distance equivalent to the square of 72.

Which one of the two children ran the larger distance?

What is the difference between the two distances?

3- An apple box weighs 10.275 kg. Calculate the mass of 10 boxes then of 10^4 boxes.

4- Put a problem whose solution is given by: 31.215×10^2.

5- Given a basin (pad) of capacity 423.15 hectoliters that is filled with water. An average of 4 *hl* of water evaporates from it every day.

a) Find the volume of water that will evaporate in 100 days. Give your answer in the form of a power.

b) What is the volume of water that will remain in the pool after 100 days?

6- Samer distributed 5 bags containing 5 marbles to each of his 5 friends

a) What is the total number of marbles? Write your answer in the form of a power of exponent 3.

b) Samer possesses 4 bags containing 4 marbles each. What is the number of marbles contained in his bags? Write your answer in the form of a power of base 4.

c) Give the number of marbles that Samer initialy had.

d) Samer's friend Mazen possesses marbles too. The number of his marbles is equal to the cube of 9.

Compare this number to the number of marbles Samer had.

7- Samir says: "The number of students of our school is between 100 and 1000, and is equal to the difference between a power of 10 and the square of 25".

Sana adds: "The number of students in our school is much larger! It is between 100 and 1 000, and is equal to the sum of a power of 10 and the cube of 7.

Is Sana right or wrong? why?

8- The side of a square garden is 3 500 m.

a) Give the area of this garden as the product of an integer by a power of 10.

b) A square piece of land of side 100 m was added to the garden.
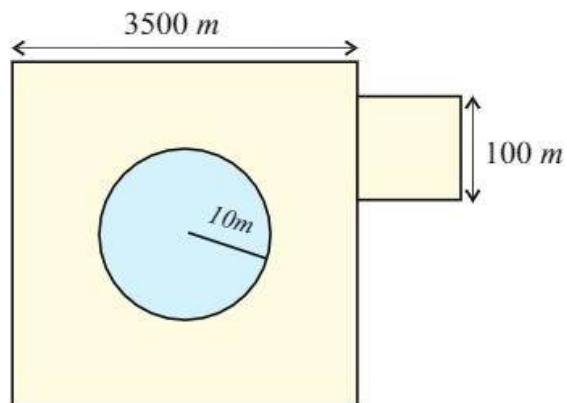
Give the area of this piece as a power of 10.

c) Write the total area as a sum of two powers of exponent 2 each.

d) At the center of this garden, a round pool of 10 m of radius was built.

Calculate the area of this pool (express it as a power).

e) Calculate the total area of the remaining piece of land.
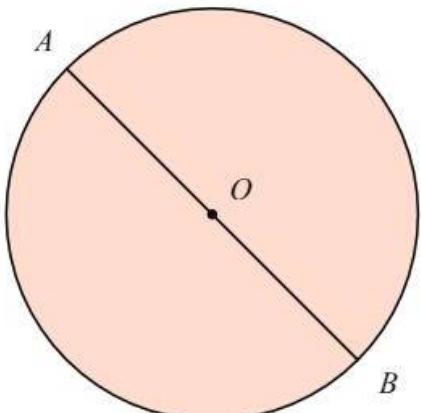

Draw a circle of center O and radius 2.5 cm
Draw the diameter $[AB]$.
Put on the circle a point C such that
$BC = 3\,cm$.
Measure the length of segment $[AC]$.
Verify that: $AC^2 + BC^2 = 25$

Adjacent angles Vertically opposite angles

7

Objectives

At the end of this chapter, I will be able to:

- identify and draw two adjacent angles; two vertically opposite angles.
- use the property of "equality" of two vertically opposite angles.
- identify complementary angles and supplementary angles.

Activities

Activity 1

| Mesure of angle $\widehat{A}$	Mesure of angle $\widehat{B}$	$\widehat{A} + \widehat{B}$
23°	67°	90°
23°	157°	
120°	60°	
	58°	90°
115°		180°
16°		90°
41°	45°	
75°	80°	

Samar found among her older brother's school materials, an old math notebook. On one of its pages, there was an incomplete table of angles. Samar tried to complete the table. Help her do so, then:

a) Copy and complete the following two sentences:
- In three cases, the sum of angles $\widehat{A}$ and $\widehat{B}$ is equal to ....
- In the other three cases, the sum of angles $\widehat{A}$ and $\widehat{B}$ is equal to ....

b) In the final two cases, how does the sum of $\widehat{A}$ and $\widehat{B}$ differ from the other sums?

Activity 2

I- On the next page, Samar saw the opposite figures on the right.

a) Observe these figures then fill in the following table by "YES" or "NO".
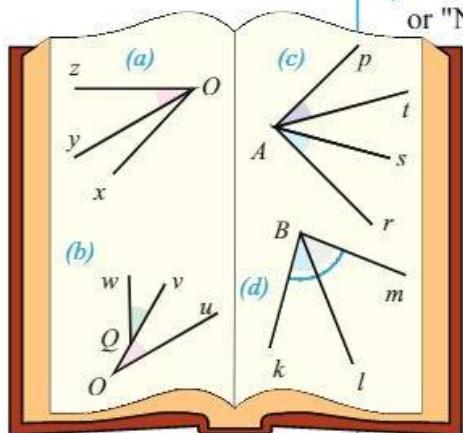
	Angles having the same vertex	Angles having a common side	Angles lying from one side and the other of the common side
(a)			
(b)			
(c)			
(d)			

b) Write an "ID" (identification card) for the pair of angles $x\widehat{O}y$ and $y\widehat{O}z$ of the figure (a).

II- The opposite figures represent a third page of this same notebook.

a) Observe these figures, then fill in the following table by "YES" or "NO".
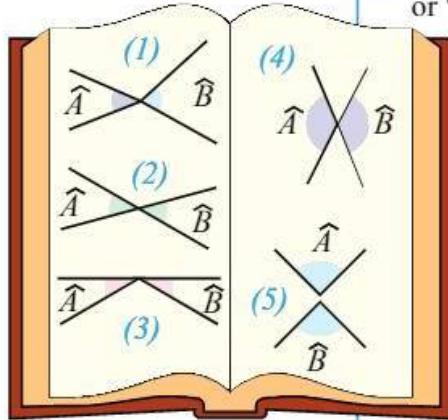
	Angles $\widehat{A}$ and $\widehat{B}$ have a common side	The sides of angles $\widehat{A}$ and $\widehat{B}$ are the prolongation of one other
(1)		
(2)		
(3)		
(4)		
(5)		

b) The angles $\widehat{A}$ and $\widehat{B}$ have the same "*ID*" in two figures. What are these figures?

c) Using your protractor, measure the angle $\widehat{A}$ and $\widehat{B}$ in the figures (2) and (4). What do you notice?

> I am an angle. My vertex is o; my sides are [ox) and [oy). They measure me by using a protactor.
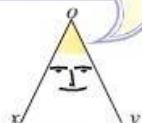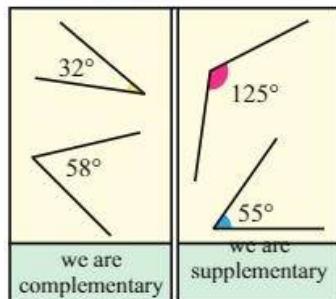
Text

There are two particularities for a pair of angles.

1) In terms of their sum, they are said to be:
- **complementary** if their sum is 90° .
- **supplementary** if their sum is 180° .

2) In terms of their position:
- They are **adjacent** when:
  - they have the same vertex.
  - they have a common side.
  - they lie on either part of their common side.

- They are **vertically opposite** when:
  - they have same vertex.
  - their sides are the prolongation of one another.
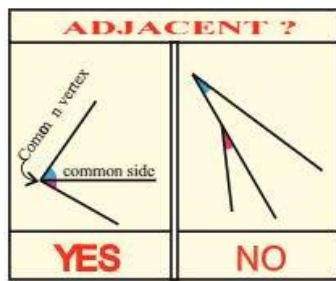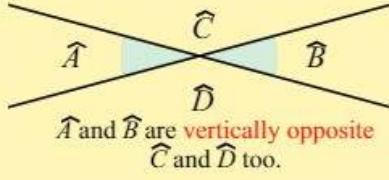
$\widehat{A}$ and $\widehat{B}$ are **vertically opposite**
$\widehat{C}$ and $\widehat{D}$ too.
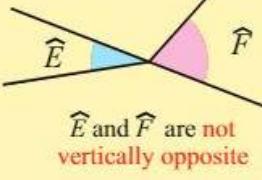
$\widehat{E}$ and $\widehat{F}$ are **not** vertically opposite

Two vertically opposite angles are equal

| $\widehat{A} = \widehat{B}$ | $\widehat{C} = \widehat{D}$ | $\widehat{E} \neq \widehat{F}$ |

we are complementary

we are supplementary

ADJACENT ?
Common vertex
common side
YES   **NO**

Focus

- Draw an angle $x\widehat{O}z$ adjacent to $x\widehat{O}y$.
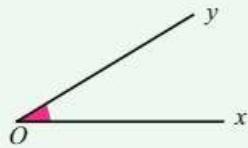
We draw the semi-straight-line $[Oz)$ for $[Oy)$ and $[Oz)$ to be from one side and the other of $[Ox)$.
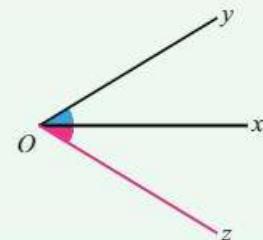
- Draw an angle $t\widehat{A}w$ vertically opposite to $u\widehat{A}v$.

|			
	We draw the semi-line $[At)$ opposite to $[Au)$ (in the prolongation of $[Au)$).	We draw the semi-line $[Aw)$ opposite to $[Av)$	$t\widehat{A}w = u\widehat{A}v$

- Two adjacent and complementary angles form a right angle.
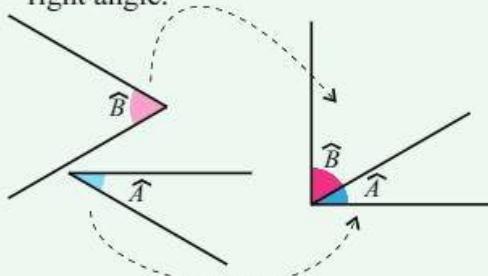
- Two adjacent and suplementary angles form a straight angle.
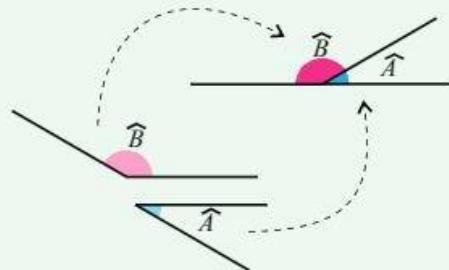
- To prove if three points are collinear:
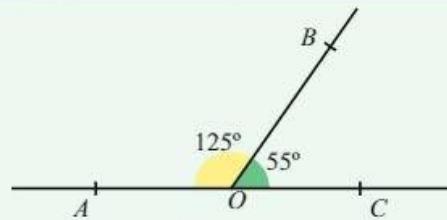
$A\widehat{O}C = A\widehat{O}B + B\widehat{O}C = 125° + 55° = 180°$.
$A\widehat{O}C$ is a straight angle.
Therefore A, O and C are collinear.
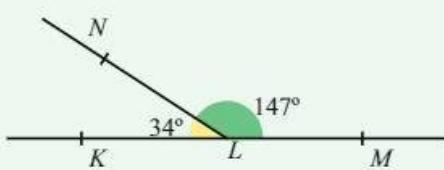
$K\widehat{L}M = K\widehat{L}N + N\widehat{L}M = 34° + 147° = 181°$.
$K\widehat{L}M$ is not a straight angle.
Therefore K, L and M are not collinear.

Exercises

Recall

| null angle	acute angle	right angle	obtuse angle	straight angle
$o \xrightarrow{y \quad x}$	acute	right	obtuse	straight

1- Observe the figure below:

a) Name an acute angle:an obtuse angle; a straight angle, a right angle and a null angle.

b) Give in degrees the measure of each of the angles above.
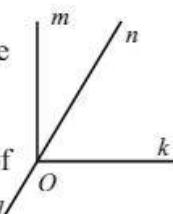
**2-** Compare the two angles in each of the 3 cases given below.

a)

b)

c)

3- Observe the following figure:
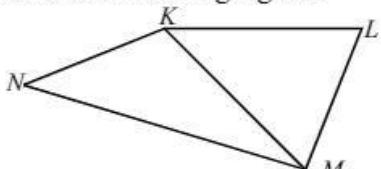
a) Are the angles $N\widehat{K}M$ and $M\widehat{K}L$ adjacent? Why or why not?
   Give another adjacent pair of angles.

b) Are the angles $N\widehat{K}M$ and $K\widehat{M}L$ adjacent? Why or why not?

4- Observe the following figure:
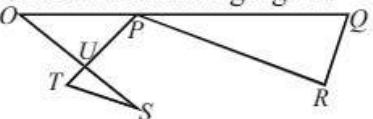
a) Are the two angles $O\widehat{U}P$ and $S\widehat{U}T$ vertically opposite? Why not?

b) Are the two angles $O\widehat{P}U$ and $R\widehat{P}Q$ vertically opposite? Why?

5- Observe the following figure:
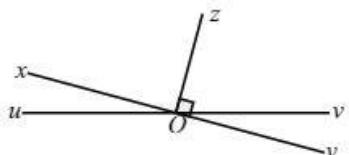
Name:
- two adjacent and complementary angles;
- two adjacent and supplementary angles;
- two vertically opposite angles;
- two complementary angles but non-adjacent.

6- Observe the figure below:
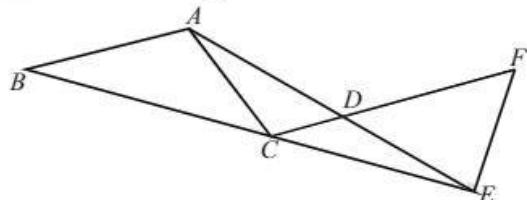
Complete the following sentences:
- $B\widehat{A}C$ and $C\widehat{A}D$ are -----------------.
- $C\widehat{E}D$ and $D\widehat{E}F$ are -----------and ------------.
- $A\widehat{D}C$ and $E\widehat{D}F$ are -------------------------.
- $B\widehat{C}A$ and $A\widehat{C}E$ are -----------and ------------.
- $C\widehat{D}A$ and $A\widehat{D}F$ are -----------and ------------.
- $A\widehat{E}F$ and $D\widehat{E}F$ are -------------------.

7- Observe the following figures:
a) Which of these 3 figures represent two vertically opposite angle?
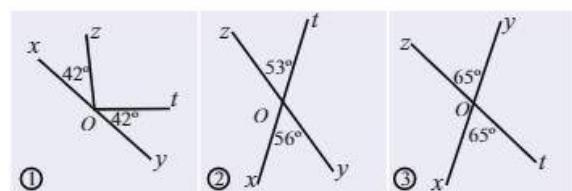
b) Why don't the other two figures represent vertically opposite angles?

8- From the following angles, choose those that form a pair of complementary angles and those that form a pair of supplementary angles.

$\widehat{A} = 28°$ $\widehat{B} = 62°$ $\widehat{C} = 133°$

$\widehat{D} = 43°$ $\widehat{E} = 152°$ $\widehat{F} = 47°$

9- In the figure given below, the lines (xy) and (zt) intersect at O.
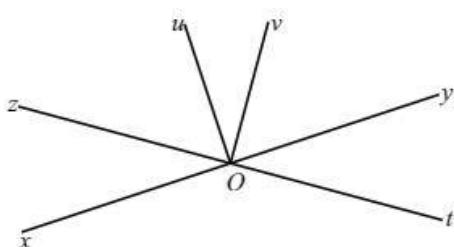
a) Name two pairs of vertically opposite angles.

b) Are $x\widehat{O}t$ and $u\widehat{O}v$ vertically opposite? Why or why not?

c) Draw a figure identical to the one above where $x\widehat{O}z = 25°$ and $u\widehat{O}v = 75°$.

d) Draw the semi-lines $[om)$ and $[on)$ such that $u\widehat{O}v$ and $n\widehat{O}m$ are vertically opposite. What is the measure of $n\widehat{O}m$?

10- a) Copy and complete following sentence:
"The angles $r\widehat{P}s$ and $s\widehat{P}t$ are .... and .... "
b) Calculate angle $s\widehat{P}t$.
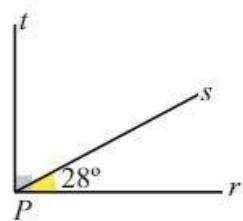
11- a) Copy and complete the following sentence:

"The angles $x\widehat{O}z$ and $z\widehat{O}y$ are .... And .... "

b) Calculate angle $y\widehat{O}z$.
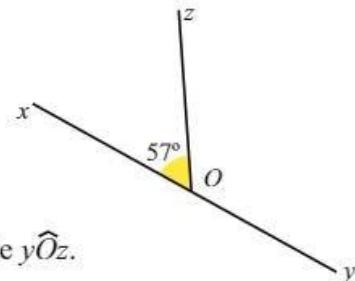
12- Reproduce the following figure and write the measure of the missing angles.
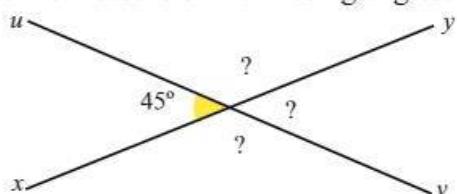
13- In each of the following figures, are the points V, I and E collinear?


Self-evaluation

1- *PARK* is a rectangle in which the diagonals intersect at O.
Name: two vertically opposite angles; two pairs of adjacent and complementary angles; two pairs of adjacent and supplementary angles; two complementary non-adjacent angles; two supplementary non-adjacent angles.

2- Construct two vertically opposite angles $v\widehat{O}t$ and $u\widehat{O}x$ such that $v\widehat{O}t = 35°$. What will the measure of $u\widehat{O}x$ be?

3- Draw an angle $x\widehat{A}y$ such that $x\widehat{A}y = 53°$.
a) Draw the angle $y\widehat{A}z$ adjacent to $x\widehat{A}y$ and such that $y\widehat{A}z = 100°$.
b) Without using a protractor, draw a complementary angle to $x\widehat{A}y$ and a supplementary angle to $x\widehat{A}y$.

Problems

1- Construct two adjacent angles $t\widehat{B}z$ and $z\widehat{B}u$ such that $t\widehat{B}z=78°$ and $z\widehat{B}u=63°$
Calculate $t\widehat{B}u$.

2- Construct two adjacent complementary angles $x\widehat{O}y$ and $y\widehat{O}z$ such that $x\widehat{O}y=48°$.
Calculate the measure of $y\widehat{O}z$.

3- Construct two adjacent supplementary angles $u\widehat{A}v$ and $v\widehat{A}w$ such that $u\widehat{A}v=108°$.
Calculate the measure of $v\widehat{A}w$.

4- Construct two supplementary equal angles. What do you call these 2 angles?

5- In each of the following figures, calculate the missing measure of the angle.

a)
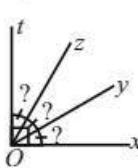
b)
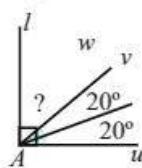
c)
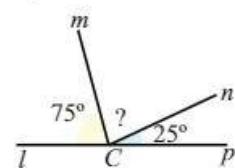
6- Observe the following figure:
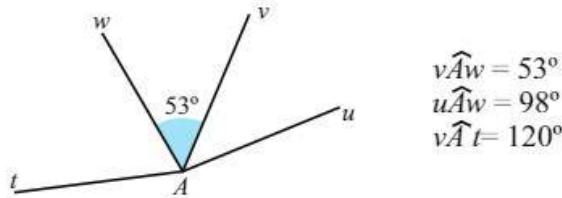
$v\widehat{A}w = 53°$
$u\widehat{A}w = 98°$
$v\widehat{A}t = 120°$

Calculate the measures of angles $u\widehat{A}v$; $t\widehat{A}w$ and tAu.

7- Construct two adjacent angles $x\widehat{O}y$ and $y\widehat{O}z$ such that $x\widehat{O}y=30°$ and $y\widehat{O}z=60°$.
Draw the semi-lines [ou), prolongation of [ox), and [ov) the prolongation of [oy).

a) What are the measures of angles $u\widehat{O}v$, $x\widehat{O}z$, $z\widehat{O}u$ and $z\widehat{O}v$?

b) Name 2 pairs of complementary angles and 2 pairs of supplementary angles (but not right angles).

c) What can you say about the semi-lines [ox) and [oz)?

8- Observe the figure below:

a) Measure using a protractor angles $A\widehat{E}O$ and $O\widehat{E}B$.
What do you notice?

b) Same question for $E\widehat{A}F$ and $E\widehat{B}F$.
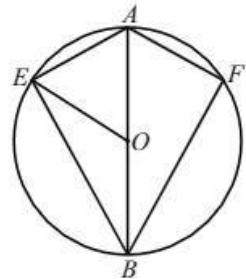
9- Draw a circle of center O and radius $r = 3$ cm.
Draw 2 diameters [RM] and [AI] such that $R\widehat{O}I=75°$.

a) Calculate the measures of angles:
$A\widehat{O}M$, $M\widehat{O}I$ and $A\widehat{O}R$.

b) Draw the quadrilateral RIMA. How are the diagonals of this quadrilateral? Why?
What is the nature of this quadrilateral?

Just for fun

Twinkle twinkle little star!!

Reproduce figures A and B.
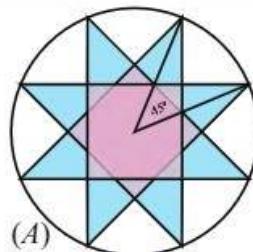
(A)
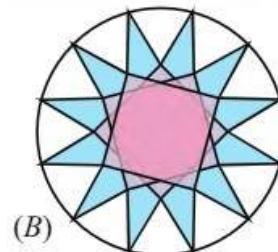
(B)

Irreducible fractions

8
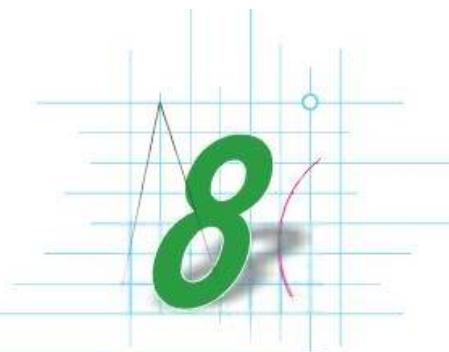
## Objectives

At the end of this chapter, I will be able to :

☐ identify an irreducible fraction ;
☐ find the irreducible form of a given fraction.

Activities
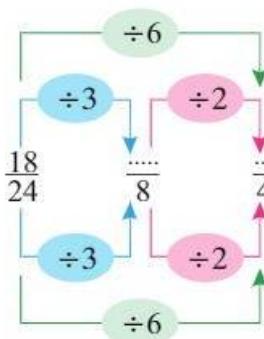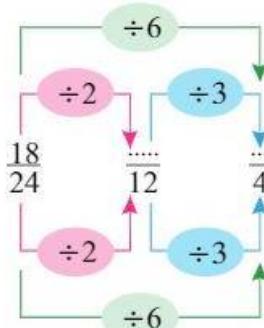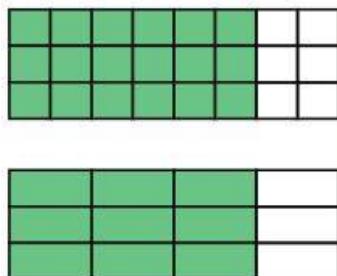
A farmer divided his rectangular field into 24 equal parts but ploughed only 18 of them.
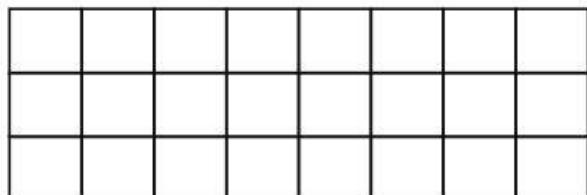
a) A child has reproduced the picture of the above rectangle representing the field, and colored 18 parts in brown.

His neighbor did the same but divided the rectangle into 12 equal parts and colored only 9 of them.

When comparing the two drawings, it was found that the colored areas were equal.

- Reproduce two equal rectangles. Divide the first one into 8 equal parts and the second into 4 equal parts. Then color in brown the parts representing the ploughed region of the field.

- Copy and complete:

  The ploughed part represents: $\dfrac{......}{24}$, or $\dfrac{......}{12}$, or $\dfrac{...}{8}$, or even $\dfrac{...}{4}$ of the field.

- Of these fractions, which is the simplest?

b) Starting from $\dfrac{18}{24}$, how can you reach $\dfrac{3}{4}$ in one step?

Do the numbers 3 and 4 have a common divisor other than 1?

What do you call these two numbers? Can you simplify $\dfrac{3}{4}$?

$\frac{30}{24}$ is reducible

$$\frac{30}{24}=\frac{30\div 3}{24\div 2}=\frac{15}{2}.$$

We simplified $\frac{30}{24}$ to $\frac{15}{12}$.

but $\frac{15}{12}$ is reducible.

$$\frac{15}{12}=\frac{15\div 3}{12\div 3}=\frac{5}{4}$$

The G.C.D of 5 and 4 is one, therefore $\frac{5}{4}$ is the irreducible fraction equal to $\frac{30}{24}$.

Criteria of divisibility :
- **by 2:** numbers ending with : 0 - 2 - 4 - 6 - 8;
- **by 5:** numbers ending with 0 or 5.
- **by 10:** numbers ending with 0.

- A fraction $\frac{a}{b}$ is said to be
  - **reducible**, if a and b have a common factor. We can then simplify this fraction.
  - **irreducible**, if a and b are prime with each other (1 is their common factor).

- To simplify a fraction is to find an equivalent fraction where the denominator and numerator have less values. Therefore, we should find the common factor to both terms of the fraction.

- To reduce a fraction is to find the irreducible fraction which is equivalent to it. This is reached:
  - either by the method of successive divisions, by finding the common divisors of the two terms of the fraction

$$\frac{48}{60}=\frac{48\div 2}{60\div 2}=\frac{24}{30}=\frac{24\div 2}{30\div 2}=\frac{12}{15}=\frac{12\div 3}{15\div 3}=\frac{4}{5};$$

  - or by the method of G.C.D:
    The divisors of 48 are: 1-2-3-4-6-8-12-16-24-48.
    The divisors of 60 are: 1-2-3-4-5-6-10-12-15-20-30-60.
    G.C.D (48 and 60) = 12

$$\frac{48}{60}=\frac{48\div 12}{60\div 12}=\frac{4}{5}.$$

Focus

reducible

$\frac{48}{60}$ We simplify by 2

$\frac{24}{30}$ reducible We simplify by 2

$\frac{12}{15}$ reducible We simplify by 3

irreducible $\frac{4}{5}$

We reduce by G.C.D: 12

Reducer of fractions !!

Criteria of divisibility:
- **by 3**: The sum of digits is divisible by 3;
- **by 9**: The sum of digits is divisible by 9;
- **by 4**: The number formed by the last two digits is divisible by 4.
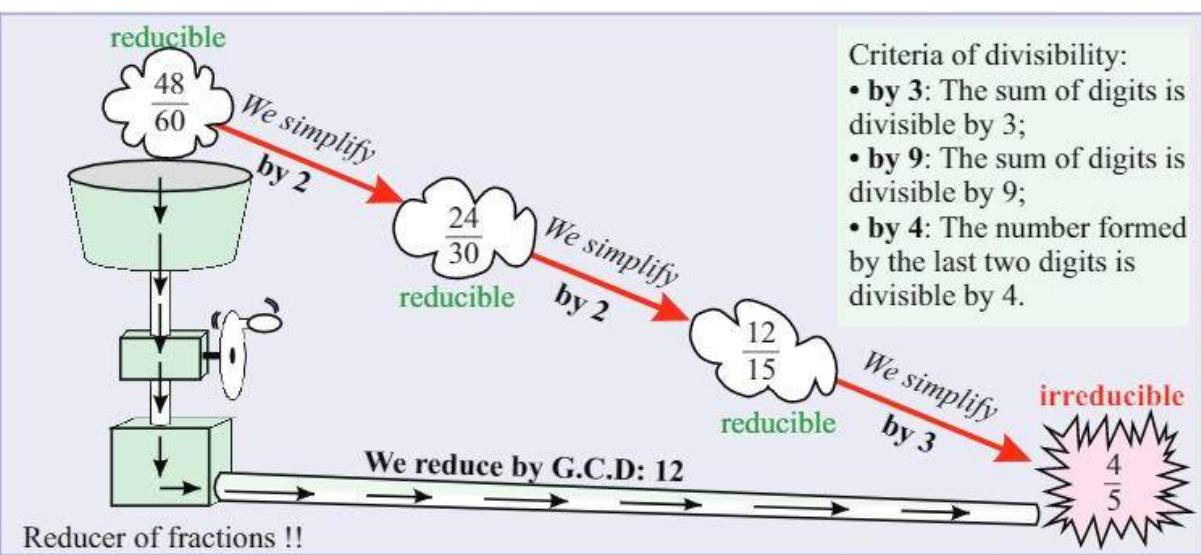
Exercises

1- Fill in the blanks:

$\frac{7}{8}=\frac{.....}{72}$; $\frac{11}{5}=\frac{.....}{35}$; $\frac{12}{7}=\frac{60}{.....}$; $\frac{24}{36}=\frac{2}{...}$; $\frac{5}{30}=\frac{1}{...}$;

$\frac{8}{72}=\frac{.....}{9}$; $\frac{123}{.....}=\frac{3}{8}$; $\frac{117}{.....}=\frac{9}{7}$; $\frac{330}{.....}=\frac{11}{2}$.

2- Find six fractions equivalent to $\frac{12}{18}$ and six other fractions to $\frac{30}{48}$.

3- Find the intruder

$\frac{21}{35}$ $\frac{180}{30}$ $\frac{270}{450}$ $\frac{12}{20}$

$\frac{39}{65}$ $\frac{3}{5}$ $\frac{24}{40}$ $\frac{210}{350}$

4- From the following fractions, frame the irreducible ones and circle the reducible ones.

$\frac{6}{9}$; $\frac{5}{5}$; $\frac{3}{17}$; $\frac{4}{7}$; $\frac{13}{29}$; $\frac{4}{9}$; $\frac{111}{423}$; $\frac{45}{12}$.

5- Complete the following equalities:

$\frac{45}{75}=\frac{9}{...}=\frac{...}{5}$; $\frac{48}{72}=\frac{12}{.....}=\frac{...}{9}=\frac{2}{...}$.

$\frac{84}{108}=\frac{.....}{54}=\frac{21}{.....}=\frac{...}{9}$.

6- Reduce the fraction $\frac{90}{126}$ using the method of successive divisions.

7- Find the irreducible fraction equivalent to each of the following fractions:

$\frac{40}{24}$; $\frac{49}{35}$; $\frac{200}{700}$; $\frac{64}{24}$; $\frac{36}{45}$; $\frac{66}{55}$.

8- a) Calculate the G.C.D of 144 and 312, then use the G.C.D to reduce the fraction $\frac{144}{312}$

b) Repeat the same procedures to reduce the fractions $\frac{81}{135}$ and $\frac{513}{180}$.

9- Find the irreducible fraction equivalent to each of the following fractions

$\frac{60}{24}$; $\frac{840}{490}$; $\frac{144}{108}$; $\frac{1400}{1050}$.

10- a) Complete the following equality $\frac{26}{65}=\frac{...}{5}$

b) Find a fraction equivalent to $\frac{26}{65}$ and which denominator is 100.

11- Draw a segment [AB] of length 8 cm and divide it into 4 equal segments.
On [AB], mark the point I such that the length of [AI] is $\frac{24}{32}$ that of [AB].

Self-evaluation

1- Indicate the irreducible fractions from the following:

$\frac{9}{7}$; $\frac{20}{65}$; $\frac{4}{16}$; $\frac{13}{15}$; $\frac{4}{13}$; $\frac{7}{28}$; $\frac{19}{3}$; $\frac{11}{121}$.

2- Reduce using both methods (successive divisions and G.C.D):

$\frac{54}{42}$; $\frac{42}{96}$; $\frac{60}{195}$; $\frac{72}{468}$; $\frac{312}{26}$; $\frac{315}{280}$.

Problems

1- Write all the fractions that are equivalent to $\frac{3}{7}$ and that have a denominator inferior to 70.

2- a) Find the irreducible fraction equivalent to $\frac{60}{135}$.

b) Find the fraction that is equivalent to $\frac{60}{135}$ knowing that the sum of its terms is 65.

3- We entered the following fractions into "a reducing machine"

$$\frac{12\,000}{8000} ; \frac{7\,500}{3500} ; \frac{270}{126} ; \frac{675}{495} ;$$

$$\frac{81}{270} ; \frac{480}{495} ; \frac{375}{525}.$$

Write the irreducible fractions that will come out. Copy and fill in the following table:

Fraction coming out	$\frac{12\,000}{8000}$	$\frac{7\,500}{3500}$	$\frac{270}{126}$	$\frac{675}{495}$	$\frac{81}{270}$	$\frac{480}{495}$	$\frac{375}{525}$
Fraction coming in							

4- a) What fractions of the hour represent:
12 min ? 35 min ? 40 min ?
45 min ? 50 min ?
52 min ? 58 min ?

b) A bus completes 21 times the entire tour of a city in 14 hours. Give in minutes the duration of a complete tour.

5- The area of a rectangular field is $792m^2$. Its length is $54m$.
Find its width. Give the answer in the form of an irreducible fraction.

6- An auditorium has 315 places, 180 of which are occupied. Give the fraction representing the number of places unoccupied with respect to the total number of places.

7- A library received 5 600 samples of a scholar by book Ⓐ , 2 700 of a book Ⓑ and 4 200 of a book Ⓒ .
The library sold 4 800 samples of Ⓐ 1 350 of Ⓑ and 3 000 of Ⓒ .

a) Give the fraction that represents the number of books Ⓐ sold with respect to all samples of Ⓐ .

b) Same question for Ⓑ and Ⓒ .

c) What fraction of the total number of books do the samples sold of book Ⓐ represent? Of book Ⓑ ? Of book Ⓒ ?

8- Walid and his parents want to go from Tripoli to Sidon. They take the coastal road as indicated below:

What fraction of the road do they complete when they reach :
a) Batroun?
b) Jbeil?
c) Beirut?

Just for fun

Find the irreducible fractions equivalent to:

$$\frac{333\,333}{888\,888} ; \frac{2\,121}{4\,242} ; \frac{123\,123}{369\,369}.$$

Bisector of an angle

9
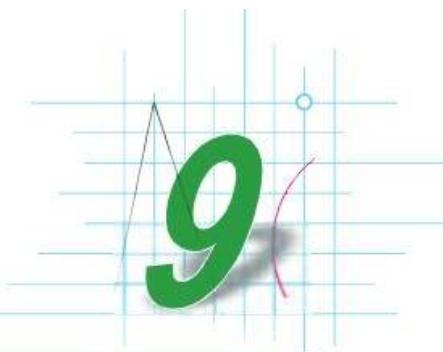
Objectives

At the end of this chapter. I will be able to:

- Recognize the bisector as an axis of symmetry of an angle.
- Construct the bisector of an angle with a protractor.
- Construct the bisector of an angle with the compass.
- Use the following property: the bisector of an angle divides it into two equal angles.

Activities
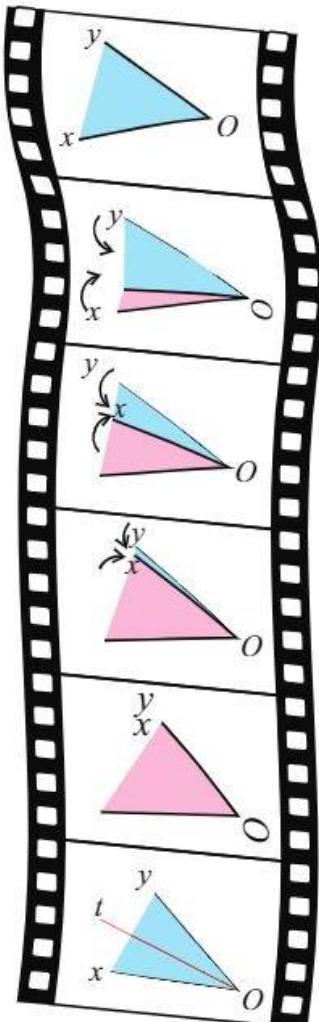
a) Draw an angle $x\widehat{O}y = 50°$ on plain paper.
Cut it and make sure that you paste (stick) both sides $[Ox)$ and $[Oy)$ by folding.

b) Open it and color the fold $[Ot)$ in red
What does $[Ot)$ represent to angle $x\widehat{O}y$?
What is the measure of both angles $x\widehat{O}t$ and $t\widehat{O}y$?

c) Draw a point I in angle $x\widehat{O}y$, following these steps:
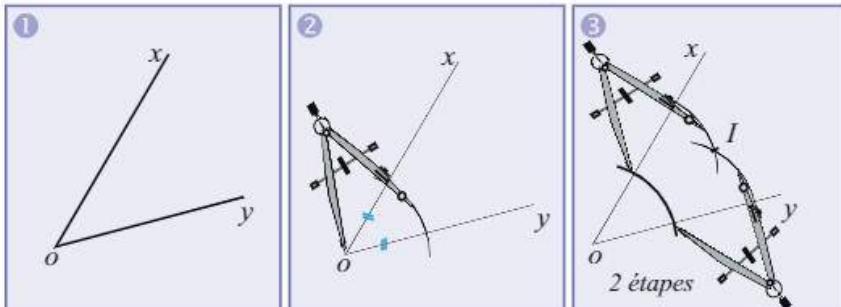
Where is the point I situated?

d) Copy and complete the following passage:
Both angles $x\widehat{O}t$ and $t\widehat{O}y$

- have the same .... ,
- have .... $[Ot)$,
- they are on the opposite side of ....

$x\widehat{O}t$ and $t\widehat{O}y$ are .....

On the other hand , $x\widehat{O}t$ and $t\widehat{O}y$ have the same .... :

$$x\widehat{O}t = t\widehat{O}y = \frac{..........°}{2} = ...°$$

The semi straight light line $[Ot)$ cuts the angle $x\widehat{O}y$ into two angular sectors .... and ....

Bisector of uÂv
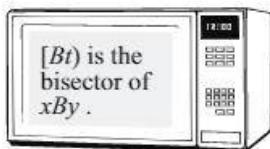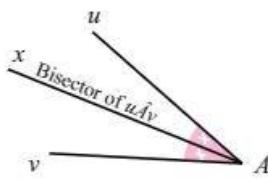
- **Angular Bisector:** Is the semi-straight line that divides an angle into two equal and adjacent angular sectors

$$u\widehat{A}x = x\widehat{A}v = u\widehat{A}v \div 2$$

- The angular bisector is the axis of symmetry of this angle.

- The angular bisector could be constructed either with a compass or with a protractor..

Bisector $u\widehat{A}v$ definitely passes through A!

[Bt) is the bisector of $x\widehat{B}y$.

- Constructing bisector [BM) of $x\widehat{B}y$ with protractor

	$110° \div 2 = 55°$	
Measure the angle $x\widehat{B}y$, $x\widehat{B}y = 110°$	Divide this measure by 2	Draw [BI) such that $x\widehat{B}t = 55°$

Construct [AD) the angular bisector $\widehat{HAO}$ using a compass: $\widehat{HAO} = 62°$
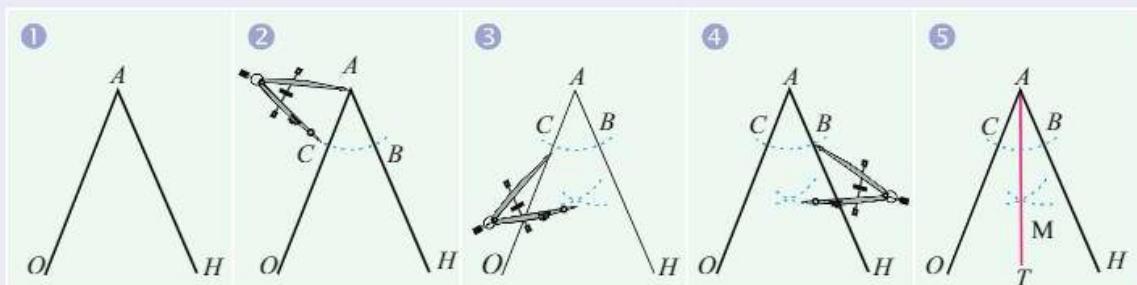
Draw an arc whose center A and radius unspecified which cuts [AH) at point B and [AO) at point C

Draw two arcs having the same radius:
The center of the first is B and the center of the second is C.

The two radii intersect at point M.
Draw a semi-line [AT) that passes through point M.

[AT) is the angular bisector of $\widehat{HAO}$: $\widehat{HAT} = \widehat{TAO} = 31°$

Exercises

1- Draw 2 angles $y\widehat{A}O$ and $O\widehat{B}t$ that measure 74º and 126º respectively. Then construct their bisectors using the ruler and the compass.

2- a) Draw an angle $C\widehat{A}S = 52º$.
b) Construct $[AT)$ bisector of $C\widehat{A}S$.
c) Copy and complete the following sentence
$$C\widehat{A}T = ...= C\widehat{A}S \div 2$$
$$= ....º \div 2$$
$$= ....º$$

**3-** Observe the following figure: $\boxed{O\widehat{A}Y = 88º}$
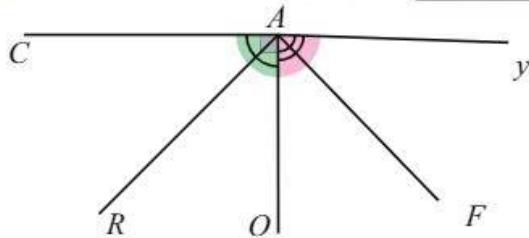
and tell whether the following statements are true or false. Justify your answer:
- $[AF)$ bisector of $Y\widehat{A}F$.
- $[AO)$ bisector of $F\widehat{A}R$.
- $[AR)$ bisector of $O\widehat{A}C$.

- measure: $Y\widehat{A}R = 90º$.
$\quad\quad\quad\quad F\widehat{A}R = 90º$.
$\quad\quad\quad\quad Y\widehat{A}O = 45º$.
$\quad\quad\quad\quad R\widehat{A}O = 45º$.
$\quad\quad\quad\quad F\widehat{A}R = 89º$.
- OAC is a right angle

4- Observe the following figure:

a) What does $[By)$ represent for angle $x\widehat{B}z$?

b) What can you say about both angles $y\widehat{B}z$ and $z\widehat{B}t$?
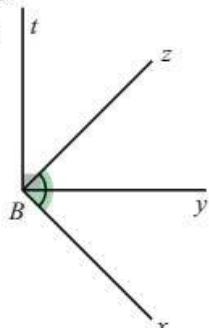
$\boxed{x\widehat{B}z = 72º}$

**5-** Observe the figure on the right:

a) What does $[OR)$ represent for angle $C\widehat{O}A$? Why?

b) Calculate the measure of angles: $C\widehat{O}R$, $A\widehat{O}L$ and $C\widehat{O}L$.

Self-evaluation

1- Copy the following figure:Identify in each case if the perpendicular bisector $[MO)$ is the angular bisector of $A\widehat{O}B$.
When it is not, draw in green the angular bisector of $A\widehat{O}B$.

2- Draw an angle $u\widehat{A}v = 48º$.
Using a compass and a ruler, construct the bisector $[AT)$ of $u\widehat{A}v$.
What is the measure of $u\widehat{A}t$? and $t\widehat{A}v$?
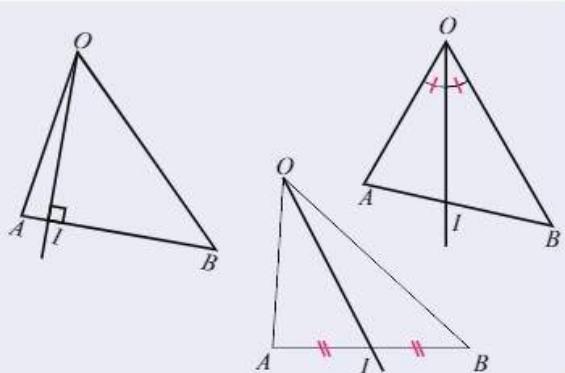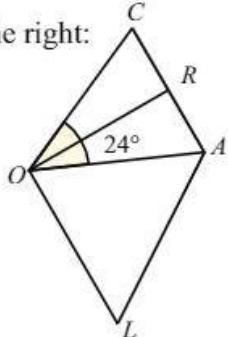
Problems

1- Observe the figure below:
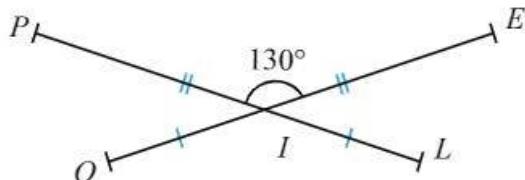
a) Redraw the figure.

b) What can you say about angle $O\widehat{I}L$ and $P\widehat{I}E$? Specify the measure of $O\widehat{I}L$?

c) Construct using a ruler and a compass the axis of the figure above.

d) What does the axis represent to $O\widehat{I}L$ and $P\widehat{I}E$?

2- Draw a parallelogram *WADI*.

a) Measure the 2 angles $\widehat{W}$ and $\widehat{D}$. Note your observations.

b) Repeat the process for both angles $\widehat{A}$ and $\widehat{I}$.

c) Draw angular bisectors of all angles *WADI*.

d) What kind of quadrilateral is formed by these bisectors?

3- a) Draw a square, a rhombus, a rectangle and a parallelogram.

b) Specify the cases in which the diagonals are the angular bisectors of these sides.

4- Draw the rectangle *CAFY* and construct the 4 angular bisectors of its sides.
What kind of quadrilateral is formed by these bisectors?

5- a) Draw two adjacent angles $u\widehat{A}v$ and $v\widehat{A}w$ such that $u\widehat{A}v = 32°$ and $v\widehat{A}w = 148°$.

b) What can you say about the two angles $u\widehat{A}v$ and $v\widehat{A}w$

c) Draw $[AC)$ and $[AS)$ the bisectors of $u\widehat{A}v$ and $v\widehat{A}w$.

d) Calculate the measure of $v\widehat{A}s$ and $u\widehat{A}s$. Note your observations for $[AC)$ and $[AS)$.

e) Copy and complete the following statement: Angular bisectors .... and .... Are ....

6- a) Draw rectangle *ABCD* of length $= 7\ cm$ and width $= 3\ cm$.

b) Draw $[DS)$ such that $C\widehat{D}S=40°$ and the angles $C\widehat{D}S$ and $S\widehat{D}A$ are adjacent. What is the measure of $S\widehat{D}A$?

c) Draw $[DH)$ such that $S\widehat{D}A=100°$ and $A\widehat{D}H$ is an acute angle.

d) Prove that $[DA)$ is the bisector of $S\widehat{D}H$.

At every green point, you have to take a turn as per the specified angle:
If the angle is preceded by a "*d*", you must turn right; if the angle is precede by a "*g*", you must turn left.

   g 45°; *d* 90°; *g* 90°;
   g 90°;*d* 90°; *d* 45°;
   d 45°; *g* 90°."

Draw the sea way that leads the sailor to the treasure.
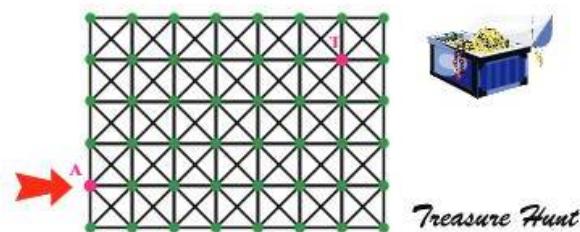
Treasure Hunt

Decimal fractions - Fractional writing of a decimal number

10

Objectives

At the end of this chapter, I will be able to:

- recognize a decimal fraction.
- write a decimal fraction in the form of a fraction whose denominator is a power of 10.
- write a decimal fraction in the form of a decimal number and vice-versa.

Activities

Rules of the game

In order to get a boat, we must find first the quotient of a by b: $q = a \div b$.
- *If $q \times b = a$, then we take a boat.*
- *If $q \times b \neq a$, then we cannot take the boat.*

Five children went on a sea trip. Each had a fraction $\dfrac{a}{b}$ on his bathing-suit. To take a boat, they should respect the rules of the game given below.

25×4=100   25×40=1000
50×2=100   250×4=1000
20×5=100   125×8=1000
2×5=10   500×2=1000
            200×5=1000

The beautiful products.

The power of 10!

$10^0 = 1$
$10^1 = 10$
$10^2 = 100$
$10^3 = 1000$

a) Copy and complete the following table:

Fraction $\dfrac{a}{b}$	Quotient of a by b	$q \times b$	Can the child take the boat?
$\dfrac{1}{5}$	$1 \div 5 = 0.2$	$0.2 \times 5 = 1$	The child will take the boat having the number 0.2
$\dfrac{45}{60}$			
$\dfrac{27}{3}$			
$\dfrac{7}{33}$			
$\dfrac{34}{125}$			

One of the children will not go on the trip. Which one and why?

b) Complete:

$$\frac{1}{5} = \frac{.....}{10} \qquad \frac{27}{3} = \frac{.....}{1} \qquad \frac{34}{125} = \frac{.....}{1000} \qquad \frac{45}{60} = \frac{3}{.....} = \frac{.....}{100}$$

Try to find a fraction equal to $\dfrac{7}{33}$ and whose denominator is a power of ten. Did you succeed?

Text

$\dfrac{12}{5}$ is a decimal fraction because:

$$\dfrac{12}{5} = \dfrac{12 \times 2}{5 \times 2} = \dfrac{24}{10}$$

$$\begin{array}{r} 2.4 \\ \hline 5\,\overline{)\,12} \\ -\,10 \\ \hline 20 \\ -\,20 \\ \hline 00 \end{array}$$

$$\dfrac{12}{5} = 2.4$$

Fractional form — Decimal form

$\dfrac{5}{6}$ is not a decimal Fraction, because :

$$\begin{array}{r} 0.833.... \\ \hline 6\,\overline{)\,50} \\ -\,48 \\ \hline 20 \\ -\,18 \\ \hline 20 \\ -\,18 \\ \hline 20 \\ .... \end{array}$$

I am a decimal number:
My whole part is: 17
My decimal part is: 29

My writing is:

17.29

and

My Fractional writing is:

$$\dfrac{1729}{100}$$

- A fraction $\dfrac{a}{b}$ is decimal if:
  - it can be written as a fraction in which the denominator is a power of 10;
  - the quotient of a by b is exact.

> The decimal writing of $\dfrac{12}{5}$ is 2.4 .

- If the quotient of the division of a by b is not exact, then the fraction $\dfrac{a}{b}$ is not a decimal fraction.

> $\dfrac{5}{6}$ is not a decimal fraction.

- A decimal number can be obtained from the division of an integer by 10, or 100 or 1000, or... Therefore, it can be written as a decimal fraction

Digit of hundredth

$$2.\boxed{37} = \dfrac{237}{\boxed{100}} = \dfrac{237}{\boxed{10^2}}$$

2 digits after the period — hundred in the denominator
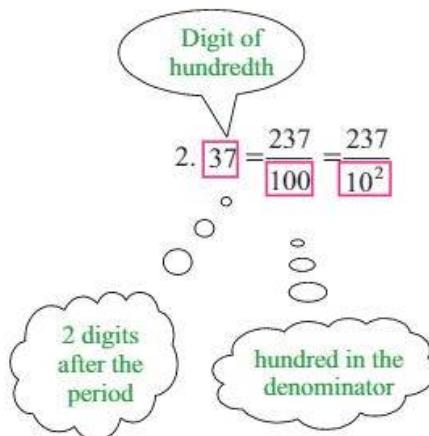
- Also, every decimal fraction represents a decimal number

$$\dfrac{49}{\boxed{10}} = 4.\boxed{9} \qquad \dfrac{1234}{\boxed{1000}} = 1.\boxed{234}$$

Decimal writing (Form)		Fractional (Form)
0,7	↔	$\dfrac{7}{10}$
4,56	↔	$\dfrac{456}{100}$
0,078	↔	$\dfrac{78}{1000}$
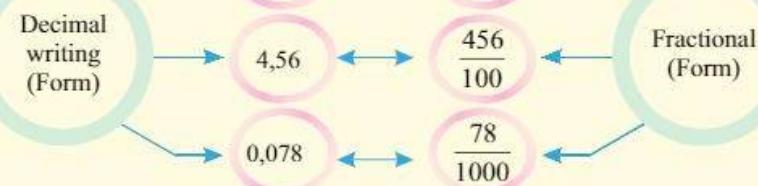
How can we identify a decimal fraction?

$\dfrac{35}{25}$

FRACTION

$\dfrac{35}{15}$

$\dfrac{7}{5}$

Irreducible fraction $\dfrac{a}{b}$

$\dfrac{7}{3}$

$\dfrac{7}{5} = \dfrac{7 \times 2}{5 \times 2} = \dfrac{14}{10}$

$\dfrac{a}{b}$ can be written as a fraction whose denominator is a power of ten.

$\dfrac{a}{b}$ cannot be written as a fraction whose denominator is a power of 10.

Impossible!!

$\dfrac{35}{25}$ is decimal

The fraction decimal

The fraction is not decimal

$\dfrac{35}{15}$ is not decimal

$\begin{array}{r} 1.4 \\ 25\overline{)35} \\ \underline{-25} \\ 100 \\ \underline{-100} \\ 0 \end{array}$

The quotient is exact

$\begin{array}{r} 2.33.... \\ 15\overline{)35} \\ \underline{-20} \\ 50 \\ \underline{-45} \\ 50 \\ \underline{-45} \\ 5 \end{array}$

The quotient is not exact

- Every decimal fraction represents a decimal number

- $\dfrac{14}{100} = 14$ hundredths $\longrightarrow$ 4 will be the digit of hundredth $\longrightarrow$ $\dfrac{14}{100} = 0.14$

- Every decimal number can be written as a decimal fraction

    $72.6 = 726$ tenths $\longrightarrow$ 6 is the digit of tenth so we write a fraction whose denominator is 10 $\longrightarrow$ $72.6 = \dfrac{726}{10}$

    $3.529 = 3\ 529$ thousandths $\longrightarrow$ 9 is the digit of thousandth so we write a fraction whose denominator is 1000 $\longrightarrow$ $3.529 = \dfrac{3\ 529}{1\ 000}$

Exercises

1- For the following quotients, circle the fractions:

$$\frac{15}{8} \; ; \; \frac{1.2}{4.5} \; ; \; \frac{28}{12} \; ; \; \frac{0.5}{6} \; ; \; \frac{19}{2.7} \; ; \; \frac{3}{50}.$$

2- Write all the following as a fraction:

$$\frac{51.7}{2.8} \; ; \; \frac{6}{0.5} \; ; \; \frac{0.24}{1.5} \; ; \; \frac{16.3}{0.88} \; ; \; \frac{3}{0.01} \; ; \; \frac{4.035}{2.5}.$$

3- a) Write all the fractions that are equivalent to $\frac{3}{4}$ and of denominator less than 45.

b) Is $\frac{3}{4}$ a decimal fraction?
Find a fraction equivalent to $\frac{3}{4}$ and whose denominator is 100.

4- Consider the fraction $\frac{12}{21}$.

a) Reduce this fraction, then find all the fractions that are equivalent to the reduced one but with a denominator less then 50.

b) Is $\frac{12}{21}$ a decimal fraction?

5- Circle the decimal fractions from the following :

$$\frac{20}{28} \; ; \; \frac{90}{100} \; ; \; \frac{6}{4} \; ; \; \frac{2}{6} \; ; \; \frac{5}{8} \; ; \; \frac{11}{3} \; ; \; \frac{6}{25} \; ;$$

$$\frac{45}{36} \; ; \; \frac{5}{24} \; ; \; \frac{24}{75}.$$

6- Find the decimal fraction that is equivalent to each of the following fractions:

$$\frac{11}{5} \; ; \; \frac{13}{50} \; ; \; \frac{63}{125} \; ; \; \frac{9}{25} \; ;$$

$$\frac{7}{8} \; ; \; \frac{47}{20} \; ; \; \frac{9}{4} \; ; \; \frac{13}{2}.$$

7- Copy and complete:

$$\frac{3.5}{100} = \frac{35}{....} = \frac{7}{....} = 0.......$$

8- Write as a decimal number:

$$\frac{7}{10} \; ; \; \frac{23}{100} \; ; \; \frac{55}{1\,000} \; ; \; \frac{37}{10} \; ; \; \frac{4\,554}{10^2} \; ; \; \frac{3\,004}{10^3}.$$

9- Write as a decimal fraction

0.8 ; 0.45 ; 0.725 ; 0.03 ; 0.015 ; 3.09 ; 5.036 ; 1.2002 ; 0.007 ; 4.001.

10- Complete as shown below:

$$\frac{3.8}{2} = \frac{38}{20} = \frac{19}{10} \; ; \; \frac{0.45}{5} = \frac{.....}{500} = \frac{.....}{100}$$

$$\frac{9.5}{5} = \frac{95}{.....} = \frac{.....}{10} \; ; \; \frac{5.6}{4} = \frac{.....}{40} = \frac{.....}{1\,000}.$$

11- Give the equivalent decimal fraction (of denominator 10) to each of the following fractions, then give the decimal number that represents

$$\frac{4}{5} \; ; \; \frac{15}{2} \; ; \; \frac{65}{50} \; ; \; \frac{48}{60} \; ; \; \frac{4.2}{2} \; ; \; \frac{18.5}{5}.$$

12- Write the decimal fraction of denominator 100 that is equivalent to each of the given fractions, then find its decimal number:

$$\frac{14}{5} \; ; \; \frac{15}{4} \; ; \; \frac{16}{50} \; ; \; \frac{43}{20} \; ; \; \frac{37}{25} \; ; \; \frac{94}{200} \; ; \; \frac{15}{500} \; ;$$

$$\frac{70}{1\,000} \; ; \; \frac{13.5}{50} \; ; \; \frac{35.2}{20}.$$

13- Write the decimal fraction of denominator 1000 that is equivalent to each of the given fractions, then find its decimal number:

$$\frac{19}{200} \; ; \; \frac{3}{500} \; ; \; \frac{17}{250} \; ; \; \frac{6}{125} \; ; \; \frac{7}{8} \; ; \; \frac{8}{25} \; ; \; \frac{45}{4} \; ;$$

$$\frac{105}{40} \; ; \; \frac{9}{5} \; ; \; \frac{69}{3\,000} \; ; \; \frac{634}{2\,000} \; ; \; \frac{18.45}{50}.$$

14- Write each of the fractions below as a decimal fraction, then give its equivalent decimal number

$$\frac{1}{4} \; ; \; \frac{2}{5} \; ; \; \frac{30}{4} \; ; \; \frac{98}{25} \; ; \; \frac{528}{125} \; ; \; \frac{72}{250}.$$

15- Match each decimal number with its fraction:

0.8 •	• $\dfrac{805}{100}$
0.145 •	• $\dfrac{9}{1\,000}$
2.3 •	• $\dfrac{48}{100}$
8.05 •	• $\dfrac{145}{1\,000}$
0.48 •	• $\dfrac{5\,275}{100}$
52.75 •	• $\dfrac{23}{10}$
0.009 •	• $\dfrac{8}{10}$

16- Give the decimal number of the following fractions :

a) $\dfrac{7}{10}$; $\dfrac{18}{1\,000}$; $\dfrac{45}{100}$; $\dfrac{65}{10}$; $\dfrac{6\,475}{1\,000}$.

b) $\dfrac{1}{2}$; $\dfrac{7}{5}$; $\dfrac{13}{4}$; $\dfrac{13}{2}$; $\dfrac{1}{8}$.

c) $\dfrac{15}{24}$; $\dfrac{175}{125}$; $\dfrac{18}{40}$; $\dfrac{44}{20}$; $\dfrac{124}{32}$.

17- Copy and complete the following equalities: $8.5 = \dfrac{.....}{10} = \dfrac{.....}{2}$.

$3.25 = \dfrac{.....}{100} = \dfrac{.....}{4}$.

$2.05 = \dfrac{.....}{100} = \dfrac{.....}{20}$.

$0.064 = \dfrac{64}{.....} = \dfrac{.....}{125}$.

18- Write each of the decimal numbers below as a decimal fraction, then reduce it if possible:
3.6 ; 0.03 ; 0.875 ; 0.72 ; 1.85 .

19- Write each of the following measures as a decimal fraction, then convert it to the wanted unit:

$45dm = \dfrac{.....}{.....} m = ...m$ $3485m = \dfrac{.....}{.....} km = ...km$

$158cm = \dfrac{.....}{.....} m = ...m$ $96m = \dfrac{.....}{.....} hm = ...hm$

$39mm = \dfrac{.....}{.....} m = ...m$ $7m = \dfrac{.....}{.....} dam = ...dam$

20- Write in the form of a decimal fraction each of the following masses, then convert it to the wanted unit:

$3dg = \dfrac{.....}{.....} g = ...g$ $634g = \dfrac{.....}{.....} kg = ...kg$

$326cg = \dfrac{.....}{.....} g = ...g$ $9g = \dfrac{.....}{.....} hg = ...hg$

$485mg = \dfrac{.....}{.....} g = ...g$ $137g = \dfrac{.....}{.....} dag = ...dag$

Self-evaluation

1- For the following fractions, find the decimal ones and give their decimal numbers:

$$\dfrac{34}{25} ; \dfrac{40}{75} ; \dfrac{12}{8} ; \dfrac{75}{35} ; \dfrac{16}{4} ; \dfrac{63}{72}.$$

2- Copy and complete the following equalities:

a) $0{,}7 = \dfrac{.....}{10} = \dfrac{70}{.....} = \dfrac{.....}{1\,000} = \dfrac{70\,000}{.........}$.

b) $5.36 = \dfrac{.....}{100} = \dfrac{.....}{1\,000} = \dfrac{.....}{10\,000} = \dfrac{536\,000}{.........}$.

c) $1.8 = \dfrac{.....}{10} = \dfrac{.....}{5}$.

d) $0.52 = \dfrac{.....}{100} = \dfrac{.....}{25}$.

e) $0.032 = \dfrac{.....}{1\,000} = \dfrac{.....}{125}$.

Problems

1- I am a decimal number. My whole part is 29 and my decimal part is 47. Write me in the form of $\dfrac{\ldots}{100}$.

2- Given the decimal fraction $\dfrac{9}{10}$. We add 10 to both terms of the fraction.

a) What is the new (obtained) fraction?

b) Is it a decimal fraction? Why or why not?

c) What is its decimal number?

3- Given the fraction $\dfrac{14}{5}$.

a) Is it a decimal fraction?

b) We add 10 to both terms of the fraction
   • What is the fraction obtained?
   • Is it a decimal fraction? Why or why not?

4- a) What does the fraction of the red rectangle represent to the square?
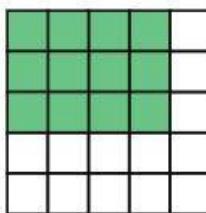
Transform this fraction into a decimal fraction.

b) Knowing that the side of the big square is 5*cm*, what is its area?

Calculate the area of the red rectangle.

5- Samer walks 4 meters in 5 regular steps. Give in fractional number then in decimal one the length of his step.

6- The distance between two cities is 200*km*. A car (A) runs 50*km*, a car (B) runs 120*km* and a car (C) runs 185*km*.

a) What is the fraction of the total distance run by (A) ? by (B) ? by (C) ?

b) In each case, write the fraction as a decimal fraction its denominator is a power of 10.

7- An amount of 75 liters of oil has to be filled in 125 equal bottles.

a) Give the capacity of a bottle as a fraction, then transform it to a fraction whose denominator is a power of 10.

b) What is in *ml* the capacity of each bottle?

8- Mirna bought 250g of cheese at 3 600L.L.

a) What is the price of one kilogram of cheese?
   Give the result as a fraction number, a decimal number, then an integer.

b) Express this sum in the form of a decimal number of thousands of L.L.

Just for fun

What fraction of the area of the square is colored? Then what fraction of the area of the rectangle is colored?
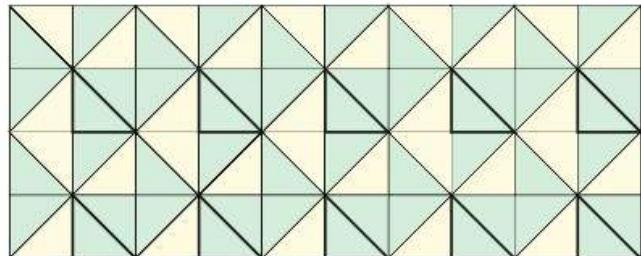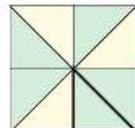
The perpendicular bisector of a segment

11

Objectives

At the end of this chapter, I will be able to:
- recognize the perpendicular bisector as an axis of symmetry of a segment;
- draw the perpendicular bisector with the set-square and the ruler;
- use the fact that every point on the perpendicular bisector is at equal distance from its extremities;
- draw the perpendicular bisector of a segment with a ruler and compasses;
- recognize the perpendicular bisector of a segment as its axis of symmetry;
- find the midpoint of a segment with the compass and the ruler.

Activities
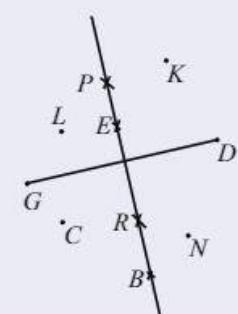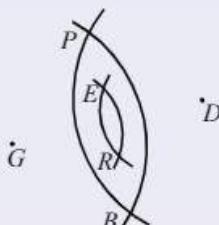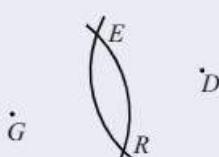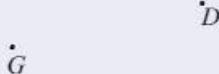
a) On a plain paper take 2 points G and D 5 cm far from each other.
Draw two of cercles having the same radius, with centers G and D; intersecting at 2 points E and R .
Construct in the same way 2 points P and B
Color in red the 4 points: P, E, R and B,
then inspect their linearity without referring to any other construction.

b) Join G and D, copy $[GD]$ and the 4 red points, then fold what you have copied in a way that G confounds with D
Open what you folded, and color the fold in green.

- What does the green line represent for $[GD]$? Does it pass through the red points?

- Make sure by using the set square and graduated ruler that the green line is perpendicular to GD and that it passes through its midpoint.

c) Observe the adjacent figure.
Measure the distances between the points R, L, P, E, K, C, N and the extremities G and D of segment $[GD]$.

Complete the following table:

$PG=.....$	$PD=.....$	$PG=PD$		$KD=.....$	$KG=.....$	$KD>KG$
$EG=.....$	$ED=.....$	$LD=.....$		$LG=.....$		
$RG=.....$	$RD=.....$	$CD=.....$		$CG=.....$		
$BG=....$	$BD=.....$	$ND=.....$		$NG=.....$		

63

(d)

I am the perpendicular bisector of [OH]

H

O

I am an axis of symetry
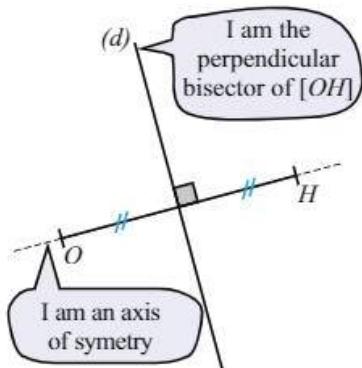
- The perpendicular bisector of a segment is the perpendicular that passes through the midpoint of this segment.
- The perpendicular bisector of a segment is the axis of symmetry of this segment.
- The characteristics of the set of points belonging to the perpendicular bisector are:
  - If a point belongs to the perpendicular bisector of a segment, **then** it is equidistant from the extremities of the segment.
  - If a point is equidistant from the extremities of a segment, then it belongs to the perpendicular bisector of this segment.
- The perpendicular bisector of a segment can be constructed either by means of a compass and a ruler, or by using a set square and a graduated ruler.
- Construction of the perpendicular bisector of *AB* using a graduated ruler and a set-square.

We Measure the length of[AB]. AB=3 cm.	Plot M, the mid.pt. of [AB].	Draw st-line(d) perpendicular to AB and passing through M.

K is closer to O than H

I am M. I am equidistant from O and H

N is equidistant from H and O

L is closer to M than O
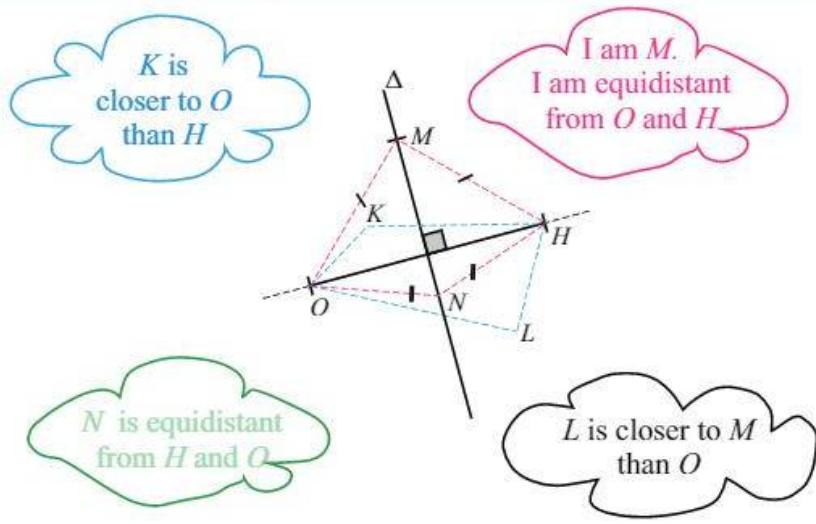
equidistant = at equal distance

■ Construction of the perpendicular bisector of $[RS]$ using a compass and a ruler.

Draw an arc with center R and radius greater than half of RS.

Draw another arc of the same radius with center S. The two arcs intersect at 2 points K and L.

Draw a line (d) that passes through K and L. This is the perpendicular bisector of $[RS]$.

So, (d) passes through the mid-point of $[RS]$. This is how we can find the mid-point of a segment using a compass and a ruler.
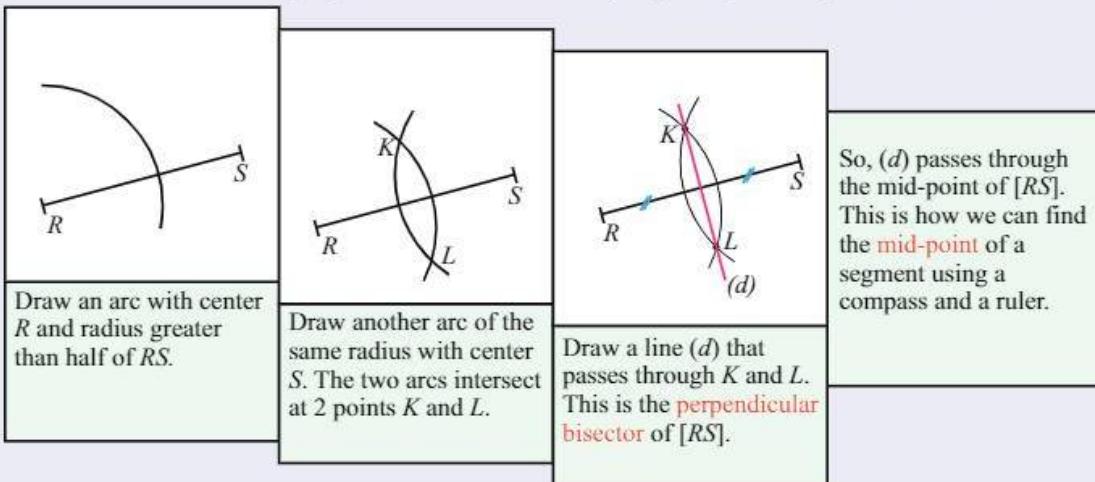
■ Characteristics of the points on to the perpendicular bisector of a segment:

M is on the perpendicular bisector of $[AB]$.

①
②

$MA=MB$
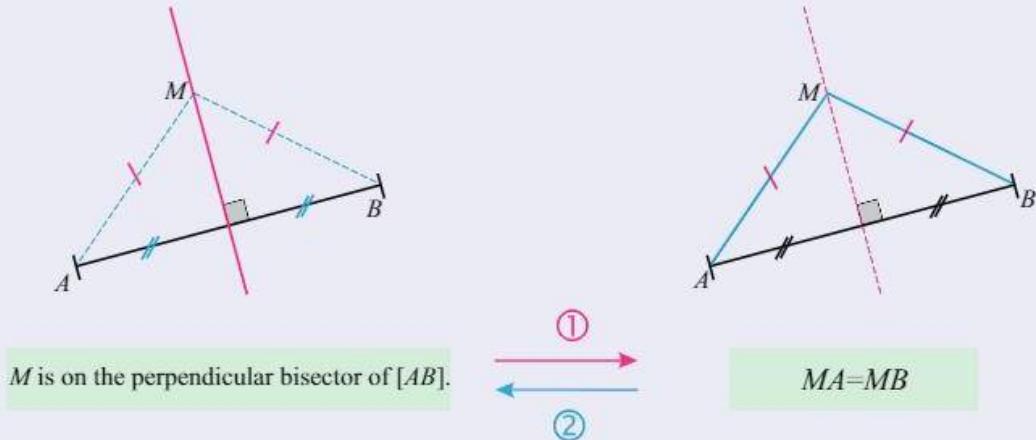
■ Partition of the plane

- (d) is the perpendicular bisector of $[GH]$: all points of (d) are at equal distance from G and H.

- The "pink" region is the region in which the points are closer to G than H.

- The "blue" region is the region of all points that are closer to H than G.
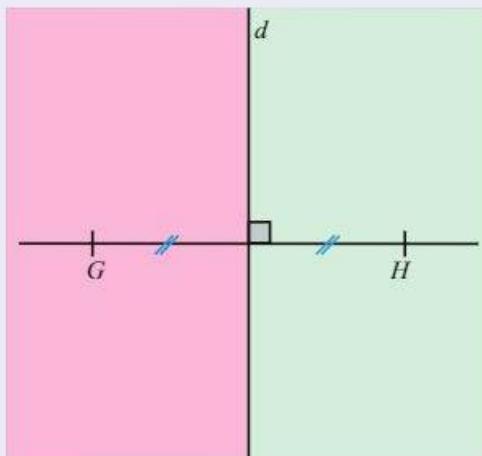
Exercises

1- Copy and complete the following sentences:

a) d_1 passes through point .... K du .... of [EF], and d_1 is .... to [GD]; d_1 is then the .... of the segment ....

b) The 2 points C and D are .... from A and B; Line (CD) is then the .... of [AB].
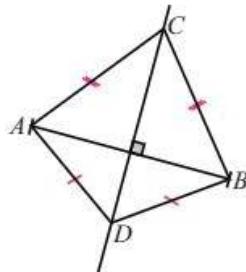
c) Line (d_2) is .... with the segment .... and passes through I which is .... from S and O. Line (d_2) is then the ....of segment ....
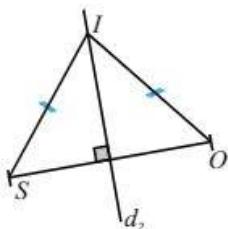
2- Draw a segment [AB] that is 7 cm in length draw the perpendicular bisector of this segment using a compass and a ruler.

3- Find using a ruler and a compass G, the midpoint of segment [TF].

$T \vdash \text{———————} \dashv F$

4- a) Draw a segment [MN] whose length is 8 cm. Take on this segment a point I such that MI=3.5 cm.

b) Draw the perpendicular bisectors of the segments [MI] and [MN].

c) How are these perpendicular bisectors?

5- a) Construct the 2 perpendicular bisectors for [RO] and [OS].

b) What is the point that is equidistant from R, O and S?
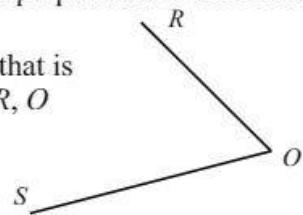
6- Observe the following figures:
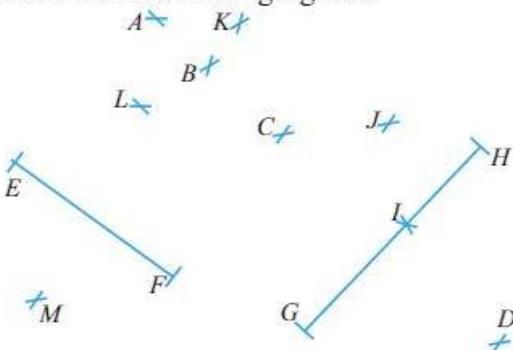
a) Using the compass only, color in:
- blue the points belonging to the perpendicular bisector [EF].
- green the points belonging to the perpendicular bisector [GH].
  Draw the two perpendicular bisectors of [EF] and [GH].

b) There is a point equidistant from E, F, G and H. Which one is it and why?

**7-** Observe the following figure:
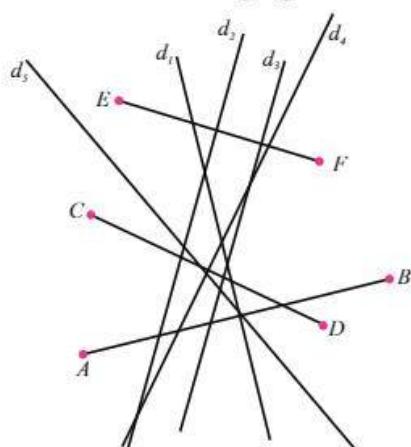
Which of the following st. lines (d_1) (d_2) (d_3) (d_4) and (d_5) is the perpendicular bisector of [AB]? Of [CD]? Of [EF]?

Self-evaluation

1- Observe the following figures:

(a)
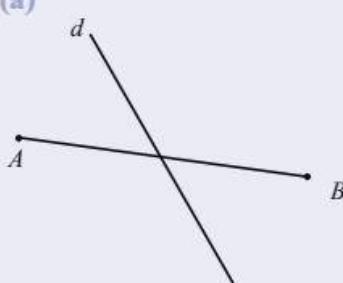
(b)
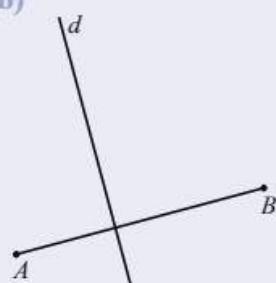
(c)
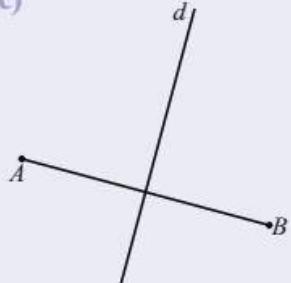
In each case point out if the line (d) is the perpendicular bisector of [AB].
Justify your answer.
Draw in green (in case it is not drawn) the perpendicular bisector of [AB].

2- Observe the opposite figure:
What does (LS) represent for [OF]?
Copy and complete the following proof:
Given: IE = IF and .... = ...
So: If a point is equidistant from the extremities of a segment,
then ....
Then: L and S are ....
The .... of OF is then ....
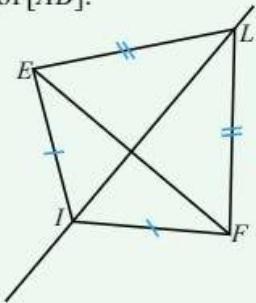
Problems

1- a) Recopy the two following figures:
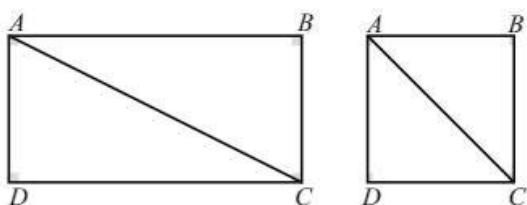
b) Draw the perpendicular bisector of the diagonal [AC] in each case.

c) Specify in each case if this perpendicular bisector passes through point B.

**2-** Draw a circle $\mathscr{C}$ of center O and radius 3 cm, and circle $\mathscr{C}'$ of center (O_1) and radius 4 cm, the two circles intersecting at M and N.

a) Show that OM = ON and that O'M = O'N.

b) What does the line (OO') represent for [MN]? Why?

3- a) Draw a circle of center O and radius 3 cm. Take two points S and F on this circle.

b) Join S and F, and draw the perpendicular bisector of the chord $[SF]$.

c) Why does this bisector pass through point O?

4- a) Write all the steps for constructing the following figure, and construct it according to the real measures below.
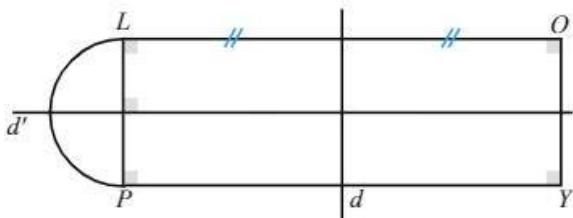

$LO = 6\ cm$
$OY = 2\ cm$

b) Which of the two lines (d) and (d') is the axis of symmetry of the figure above?

5- Color the square *RAMI* below:

• In red: the region where the points are closer to point M than point R, and at the same time closer to A than I ;
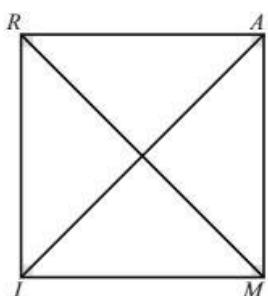
• In green: the region where the points are closer to I than A and closer to R than M ;

• In blue: the group of points equidistant from R and M.

6- We want to place a water sprinkle equidistant from the orange tree, the cherry tree and the pear tree.
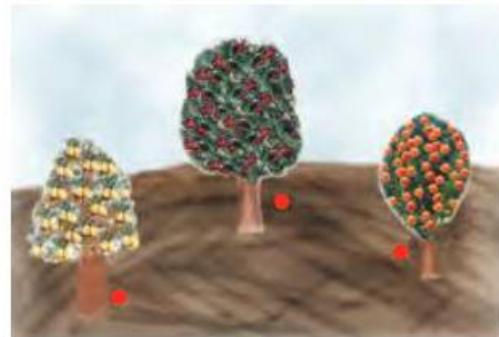
Specify the appropriate point to place this sprinkle.

- My name is Rami. I live in the same neighborhood with my friends Sami, Rima, Jihad and Wadad.

- I'm closer to Rima than Sami.

- I live equidistantly from Jihad and Wadad.

- I live 500 m away from Sami.

- Specify the place where I live.

# Triangles

12

Objectives

At the end of this chapter, I will be able to:

- define and construct bisectors, heights, medians and perpendicular bisectors in a triangle, and know that they are concurrent;
- determine the center of the circle passing thru 3 non-collinear points;
- identify particular triangles;
- know that the sum of angles in a triangle is equal to 180°.

Activities

Activity 1
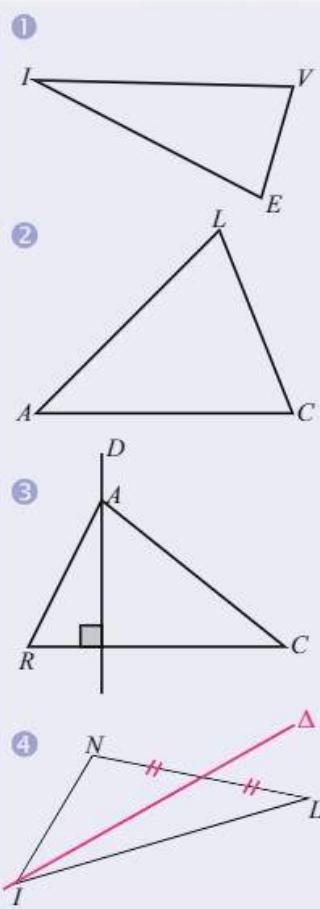
a) Reproduce triangle *VIE* shown in figure 1, and draw the bisectors of the three angle $\widehat{V}, \widehat{I}$ and $\widehat{E}$.
What can you say about these 3 bisectors?

b) • Reproduce triangle *LAC* shown on figure 2, and draw *d* and *d'*, the perpendicular bisectors of sides [*AC*] and [*LC*] respectively. Let *O* be their point of intersection.

- Draw the circle of center *O* and radius *OL*.
Does this circle pass through the other two vertices of triangle *LAC*?

- Draw the perpendicular bisector of [*AL*]. What do you notice?

c) • Observe figures 3 and 4 then complete:

- In triangle *ARC*, we drew the straight line *D* possing through the vertex .... and that is .... to the side [*RC*].

- In triangle *NIL*, we drew the line Δ passing through the vertex .... and the .... of side [*NL*].

- Reproduce the triangles in figures 3 and 4 then draw:

- in triangle *ARC*, the lines *D'* and *D"* having same characteristics as D but passing through *R* and *C* respectively.

- In triangle *NIL*, the lines Δ' and Δ" having same characteristics as Δ but passing through *I* and *L* respectively.

- What can you say about the three drawn lines in each of the two triangles?

69

Activity 2
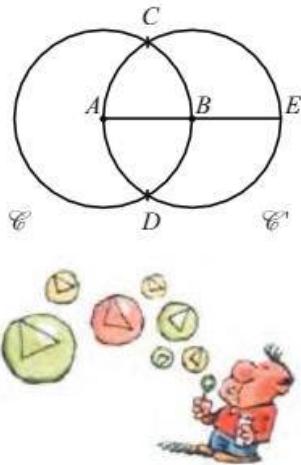
a) Locate 2 points A and B distant by 2.5 cm. Draw two circles $\mathscr{C}$ and $\mathscr{C}'$ of centers A and B and of equal radius 2.5cm cm. These two circles intersect at C and D. Draw the diameter $[AE]$ of circle $\mathscr{C}'$.

Also, draw the triangles ACD, ABC and ADE.

b) Copy and complete the following table:

Triangles	Lengths of sides	Measure of angles	Characteristics of triangle
ACD	$AC=$ $AD=$ $CD=$	$C\widehat{A}D=$ $C\widehat{D}A=$ $A\widehat{C}D=$	The sides $[AC]$ and $[AD]$ are .... The angles $A\widehat{C}D$ and $A\widehat{D}C$ are ....
ABC	$AB=$ $AC=$ $BC=$	$A\widehat{B}C=$ $A\widehat{C}B=$ $B\widehat{A}C=$	The 3 sides have the same .... The 3 angles are ....
ADE	$AD=$ $AE=$ $DE=$	$A\widehat{D}E=$ $D\widehat{A}E=$ $A\widehat{E}D=$	Angle $A\widehat{D}E$ is .... The sides $[AD]$ and $[DE]$ are ....

Activity 3

The triangle CAR has $CA=7cm$, $AR=6cm$ and $RC=5cm$.

a) Copy the angles of this triangle $\widehat{C}$, $\widehat{A}$, and $\widehat{R}$ separately, then color $\widehat{A}$ in blue, $\widehat{R}$ in green and $\widehat{C}$ in red.
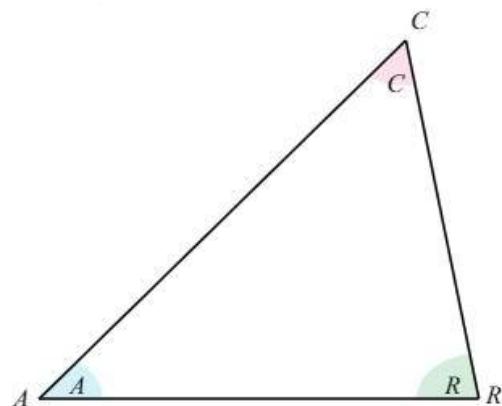
**b)** Put angle $\widehat{C}$, $\widehat{A}$, and $\widehat{R}$ in the order shown in the figure next.

What can you call the angle formed by the 3 angles of triangle CAR?
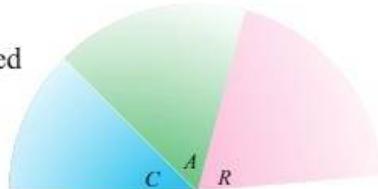
c) Complete the following equality: $\widehat{C}+\widehat{A}+\widehat{R}=$..... °

d) Verify this result by measuring the angle $\widehat{C}$, $\widehat{A}$ and $\widehat{R}$ using your protractor.

70
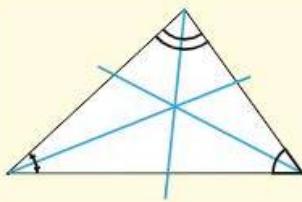
I- Particular lines in a triangle

- A **bisector** of a triangle is the bisector of an angle of this triangle. The three bisectors in a triangle are **concurrent**.

- In a triangle, the **perpendicular bisectors** of the three sides are **concurrent** in a point. This point is the center of the circle that passes through the three vertices of the triangle. The circle is called **circle circumscribed to the triangle**.
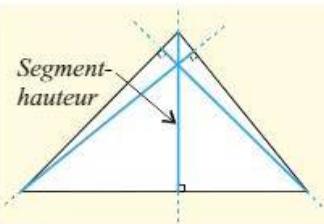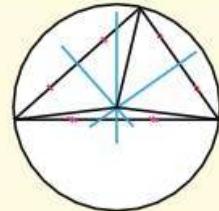
- A **height** in a triangle is the line passing through a vertex of this triangle and perpendicular to its opposite side.

> **Warning!** We call "**height**" too the height segment (part of the height passing through the triangle) or the distance from the vertex to the opposite side

- A **median** in a triangle is the line passing through a vertex of this triangle and through the mid-point of the opposite side.
- The **three heights** in a triangle are **concurrent**; the three medians are too.

II- Particular triangles

- An **isosceles triangle** is a triangle that has 2 sides of equal lengths. *ISO* is isosceles of vertex *I*. Its base is [*SO*].

- An **equilateral triangle** is a triangle that has three sides of equal lengths. *TRI* is equilateral.

- A **right triangle** is a triangle having two perpendicular sides *ABC* is right at *A*. Its hypotenuse is [*BC*].
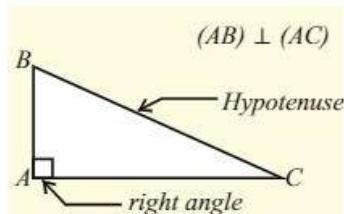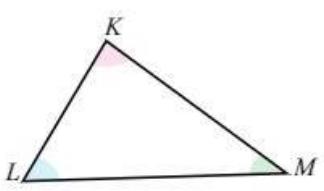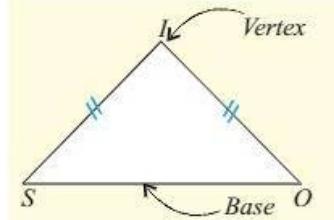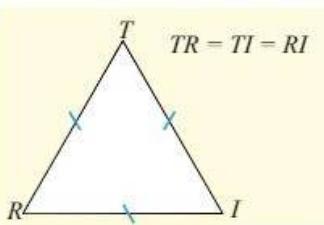
III- Sum of angles in a triangle

The sum of angles in a triangle is equal to **180°**.

$$\widehat{K}+\widehat{L}+\widehat{M}=180°.$$
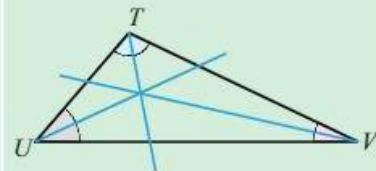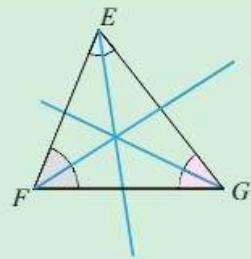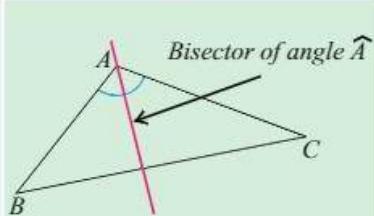
The point of intersection of the three bisectors is always inside the triangle
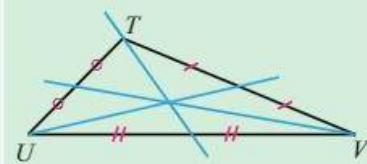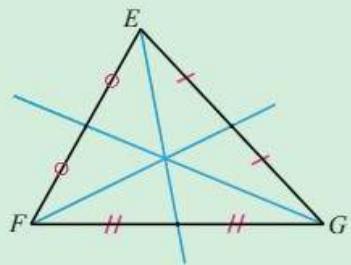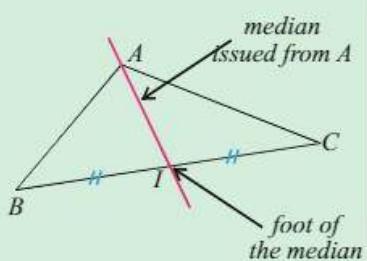
We call also "median" the segment [AI] or length AI.

The point of intersection of the three medians is always found inside the triangle
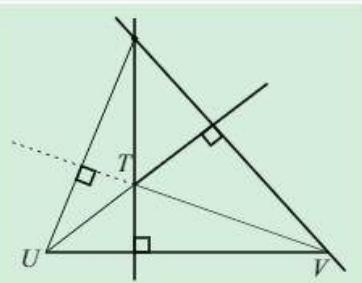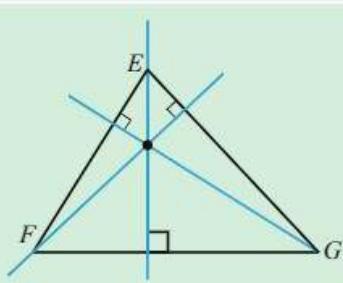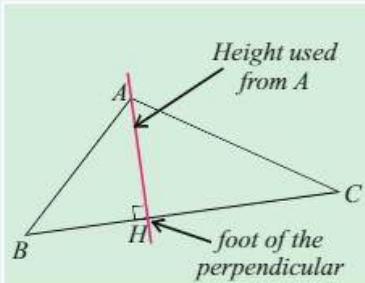
We call also "height" the segment [AH] or the length AH.

The point of intersection of the three heights is inside the triangle EFG.

The point of intersection of the three height is outside the triangle TUV.

## ☐ Construction of the circle circumscribed to a triangle
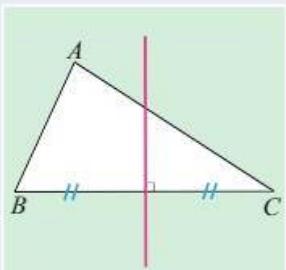
We draw the perpendicular bisector of one side
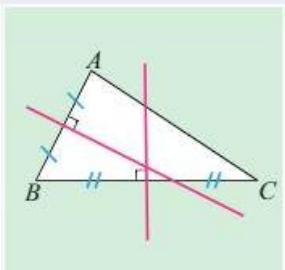
We draw the perpendicular bisector of another
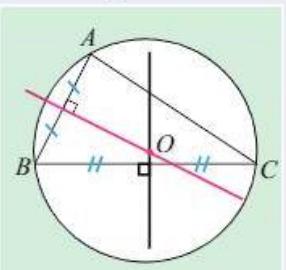
We draw the circle of center O and radius OA

LKM is obtuse
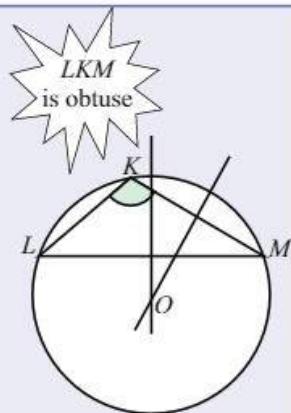
I am free of my sides but not of my angles

☐ Angles in a triangle

Any triangle	Isosceles triangle	Right triangle	Right isosceles triangle	Equilateral triangle
$\widehat{A}+\widehat{B}+\widehat{C}=180°$	$\widehat{B}=\widehat{C}$ $\widehat{A}+(2\times\widehat{B})=180°$	$\widehat{A}=90°$ $\widehat{B}+\widehat{C}=90°$	$\widehat{A}=90°$ $\widehat{B}=\widehat{C}=\dfrac{90°}{2}=45°$	$\widehat{A}=\widehat{B}=\widehat{C}=\dfrac{180°}{3}=60°$

Exercises

1- Observe the following figure:
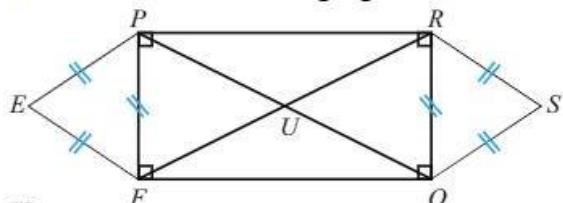
Name:
- The right triangles;
- The isosceles triangles;
- The equilateral triangles;
- A non-particular triangle.

2- Reproduce triangle *PIE* given below then draw:
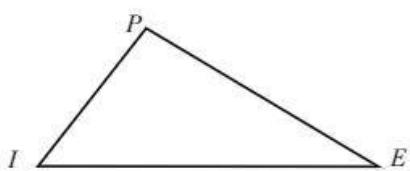
- in red, the height issued from E;
- in green, the bisector of $\widehat{PIE}$;
- in blue, the median issued from P;
- in black, the perpendicular bisector of [PI].

3- Reproduce using a ruler and a compass the triangle ABC.
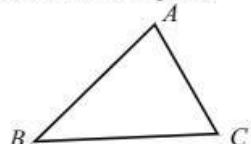
a) Draw:
  ☐ the line d_1, the height issued from A;
  ☐ the line d_2, the median relative to side [AC];
  ☐ the line d_3, the perpendicular bisector of [BC].
b) What can you tell about d_1 and d_3? Why?

4- Reproduce, using your ruler and compass the triangles given below, then construct the circles circumscribed to these triangles.
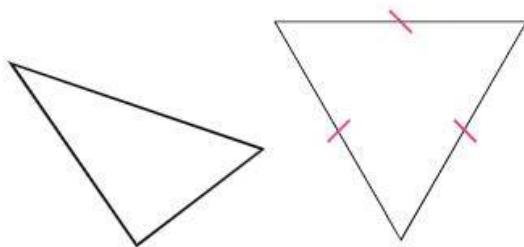
5- Construction of triangles of known side lengths.

a) **Example:** Construct a triangle *ABC* such that :

$AB = 1.8cm$; $AC = 1.3cm$ and $BC = 2.2cm$.

- We draw a segment [*BC*] of length 2.2 *cm*	- We draw an arc of center *B* and radius 1.8 *cm*.	- We draw an arc of center *C* and radius 1.3 *cm*. - *A* is one of the point of intersection of the 2 areas.
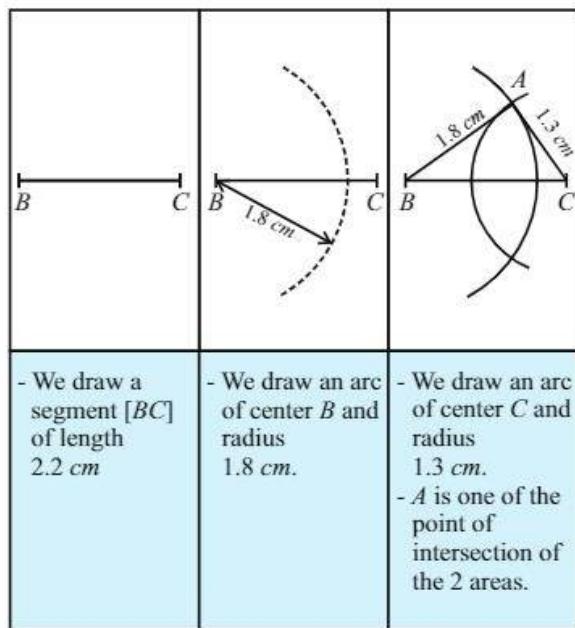
b) Construct the triangles:

- *RIZ* such that
  $RI = 8cm$; $ZR = 5cm$; $ZI = 7cm$.
- *LAC* such that
  $LC = 7.5cm$; $AL = 2cm$; $AC = 6.5cm$.
- *ELF* such that
  $EL = 4.5cm$; $LF = 7cm$; $EF = 3.5cm$.

6- Construct an isosceles triangle *LOI* of vertex *O* such that $LO=4.3cm$ and $OI=6.5cm$. What is the base of this triangle?

7- Construct an equilateral triangle *PLI* of 162 *mm* of perimeter.

8- Construct a triangle *REC*, right at *E*, such that $RE = 3.5\ cm$ and $EC = 65\ mm$.

**9- Construction of triangles of 2 known side measures and one angle.**

a) **Example:** Construct a triangle *EFG* such that: $EF = 2cm$; $EG = 1.7cm$ and $F\widehat{E}G = 40°$.

- We draw a segment *EF* of length 2 *cm*. - We draw a semi-line [*Ex*) such that $F\widehat{E}X=40°$.	- We locate on [*Ex*) the point *G* such that $EG = 1.7cm$.	- We draw the segment *GF*.

b) Construct the triangles:

- *ROC* such that $RO=6cm$, $OC=4.5cm$ and $R\widehat{O}C=75°$.
- *BAL* such that $BA=4.5cm$, $AL=7.5cm$ and $B\widehat{A}L=45°$.
- *VER* such that $VE=5cm$, $VR=8cm$ and $E\widehat{V}R=130°$

10- a) Construct a triangle *ABC* such that: $AB = 2.5cm$, $BC = 3cm$ and $CA = 3.5cm$.
b) At the exterior of this triangle, draw:
- the equilateral triangle *ABE*;
- the isosceles triangle of vertex *C* and such that $B\widehat{C}F = 50°$
- the triangle *ACG*, right at *A*, and such that $AG = 3.5cm$.

**11-** Construct the isosceles triangle *ISO* of vertex *I* and such that $SO=6cm$ and $IS=9cm$.
a) **Draw:**
- the 3 heights that intersect at *H*.
- the 3 medians that intersect at *M*.
- *O* the center of the circle circumscribed to that triangle.
b) What do you notice regarding the points *H*, *M*, *N* and *O*?

12- Construction of triangles of 2 known angles and one known side.

a) **Example:** Construct a triangle KLM such that $KL=1.5cm$; $\widehat{KLM}=65°$ and $\widehat{LKM}=40°$.

K 1.5cm L	40° K 1.5cm L	y M x 65° K 1.5cm L
- We draw a segment KL of 1.5 *cm* length.	- We draw a semi-line $[Kx)$ such that $L\widehat{K}x = 40°$.	- We draw the semi-line $[Ly)$ such that $K\widehat{L}y=60°$ and which intersects $[Kx)$ at M.

b) **Construct the triangles:**

- VAL such that $VL = 2.5cm$, $\widehat{VLA} = 52°$ and $\widehat{LVA} = 86°$.
- LIT such that $LI = 5cm$, $\widehat{LIT} = 110°$ and $\widehat{ILT} = 30°$.
- MOT such that $OT = 7cm$, $\widehat{MOT} = 35°$ and $\widehat{MTO} = 55°$.

13- Write a given to help us construct the following figure:

BO=BL=3.5cm.
OL=4cm.
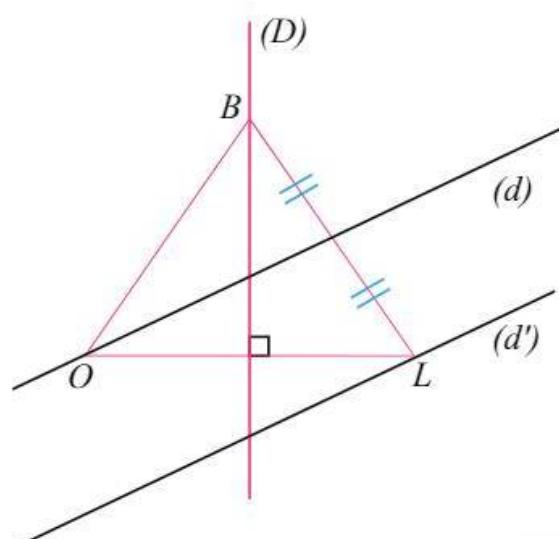
14- Reproduce an angle using a ruler and a compass.

a) **Example:** Construct an angle $u\widehat{A}v$ equal to angle $x\widehat{O}y$, using the ruler and the compass.
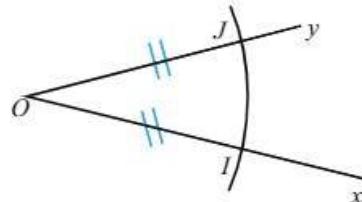
	- We draw an arc of a circle of center O, and radius r that intersects $[Ox)$ at I and $[Oy)$ at J. - We draw a semi-line $[AU)$ and an arc circle of center A and radius OI that intersects $[AU)$ at K.
	- We draw an arc of circle of center K and radius IJ that intersects the 1st one at L.
	- We draw the semi-line $[Av)$ that passes through L. - We get: $\boxed{u\widehat{A}v=x\widehat{O}y}$

b) Using your compass and a ruler, reproduce all the triangle given next:
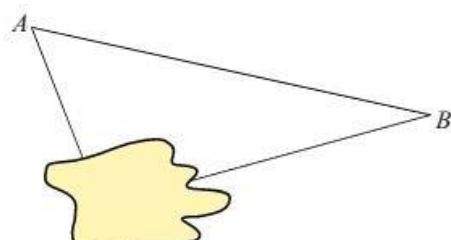
15- Draw a triangle REC such that $ER=6cm$, $EC=2.5cm$ and $\widehat{REC}=90°$.
a) What is the nature of this triangle?
b) Construct the circle circumscribed to this triangle.
c) Where is its center located at?

16- Find the missing angle in each of the following cases:

a) ABC is a scalene triangle:

$\widehat{A} = 62°$; $\widehat{B} = 38°$; $\widehat{C} =$ ☐ .

b) EFG is an isosceles triangle of vertex F:

$\widehat{F} = 54°$; $\widehat{E} =$ ☐ ; $\widehat{G} =$ ☐ .

c) KLM is an isosceles triangle of vertex L:

$\widehat{K} = 35°$; $\widehat{M} =$ ☐ ; $\widehat{L} =$ ☐ .

d) RST is a triangle right at S:

$\widehat{R} = 25°$; $\widehat{S} =$ ☐ ; $\widehat{T} =$ ☐ .

17- Is it possible to draw a triangle ABC such that:
a) $\widehat{A} = 125°$; $\widehat{B} = 25°$; $\widehat{C} = 35°$;
b) $\widehat{A} = 28°$; $\widehat{B} = 75°$; $\widehat{C} = 77°$;
c) $\widehat{A} = 45°$; $\widehat{B} = 66°$; $\widehat{C} = 70°$?

18- Draw a triangle PRE such that: $PR=5cm$; $PE=8$ cm; $R\widehat{P}E=110°$, and a triangle BOL such that: $BO= 5$ cm; $BL = 8$ cm; $O\widehat{B}L=70°$.

a) Construct the circles circumscribed to these triangles.
b) In each case, is the center of the circle inside or outside the triangle? Why or why not?

19- Construct in each of the following cases an isosceles triangle ISO of vertex I.
a) $IS=3.5cm$ and $S\widehat{I}O=115°$.
b) $IS=6cm$ and $I\widehat{S}O =50°$.

20- Construct a triangle COR right at O and such that $OR = 6$ cm, and $O\widehat{R}C=32°$. What is the measure of angle $O\widehat{C}R$?

21- Observe the following figure:

Are the points O, L and E collinear?

Self-evaluation

1- a) Observe the figure, given next, then fill in the blanks:
- In triangle PIE, (PT) is the .... issued from ....
- In triangle POT, $[PI]$ is the .... of ....
- The line (PR) is the .... issued from ....in the triangles ....; ....; .... and ....
- The line (PR) is the .... of side .... of triangle ....

b) What should we draw on the figure in order to obtain the center A of the circle circumscribed to triangle POD?

2- a) How many triangles can you see on the kite drawn next.
b) Name:
- 4 right triangles
- 3 isosceles triangles
- 1 equilateral triangle
- 2 scalene triangles.

3- In the table below, $\widehat{T}$, $\widehat{R}$ and $\widehat{I}$ are the angles of triangles TRI. Reproduce and complete the table:

$\widehat{T}$	$\widehat{R}$	$\widehat{I}$	Nature of triangle TRI
48°	54°		
	24°	66°	
60°		60°	
36°		72°	
45°	45°		
	106°	50°	

1- Draw a circle $\mathscr{C}$ of center O and radius $R = 3$ cm. Locate on this circle a point E. Draw the perpendicular bisector (d) of $[OE]$ that intersects the circle $\mathscr{C}$ in A and B.

Prove that OAE is an equilateral triangle. To do so, put in sequence the following sentences:

☐ Therefore $OE=OA=AE$.

☐ $[OA]$ and $[OE]$ are 2 radii of circle $\mathscr{C}$.

☐ A is the perpendicular bisector of $[OE]$.

☐ A and E are on the circle $\mathscr{C}$.

☐ Therefore $OE=OA$.

☐ Therefore $AE=AO$.

☐ This is an equilateral triangle.

☐ Since triangle OAE has 3 equal sides.

2- Observe the following figure:

$OP=1.5cm$
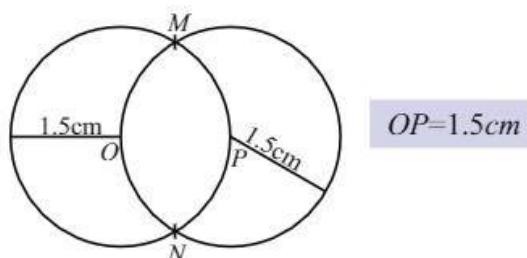
a) Reproduce the figure above and draw the triangles POM and MON.

b) What is the nature of each of these triangles? Justify your answer in each case.

3- a) Reproduce the given figure then construct the vertex G of triangle EFG such that (D) be at the same time bisector of FEG and height issued from E.

b) What is the nature of triangle EFG?
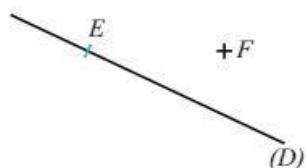
4-
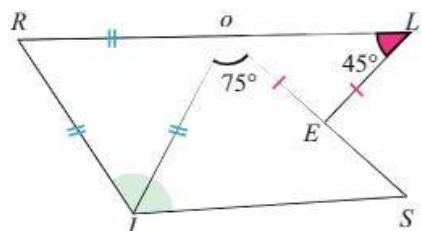
a) Look at the figure then complete the text:
- According to the figure, we have,
.... $= RO =$ ....
The triangle ROI having his 3 sides of same ...., is an .... triangle .
So, the 3 angles of this triangle are..........
and each one measures ....
- The angles $R\widehat{I}O$ and $S\widehat{I}O$ are .... and equal according to the figure, therefore angle $S\widehat{I}O$ measures ....
- The sum of angles in a triangle is equal to .... and $S\widehat{I}O =$.......... and $I\widehat{O}S=$........
Therefore, $I\widehat{S}O =$ ....
b) What is the measure of angle $L\widehat{O}E$?
c) Calculate the measure of angle $L\widehat{E}O$.
What is the nature of triangle LEO?

5- The points J, S and Z represent the houses of Jad, Samar, and Ziad respectively.

The 3 friends promise to meet at a place equidistant from J, S and Z.

Reproduce these 3 points and construct the point of their meeting.

$+J$

$S+$

$+Z$

Here is a drawing!

But there are some letters missing.

We know that:

- the triangle is called *MUR*
- the line d_1 is the median issued from R
- the line d_2 is the bisector of *RUM*.

Locate *M; U; R;* d_1*;* d_2 and d_3!

What does d_3 represent for triangle *MUR*?
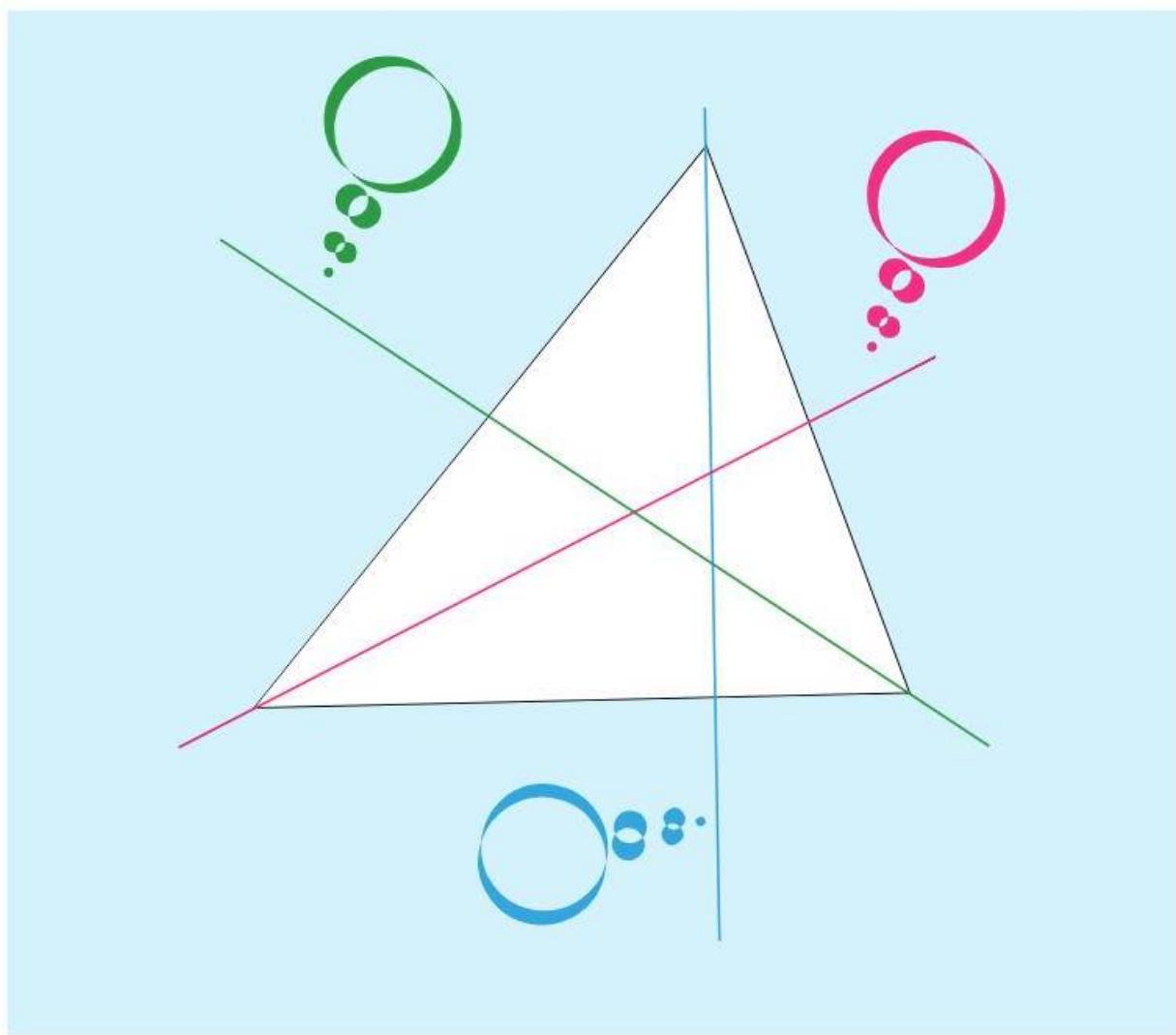
Development of a decimal number in terms of powers of 10 and $\frac{1}{10}$

13

Objectives

At the end of this chapter, I will be able to:
- ☐ Write a decimal number in terms of powers of 10 and $\frac{1}{10}$.
- ☐ Approximate (round) a decimal number.

Activities

Graduation
blue → units
red → corresponding → tenths
green → to → hundredth
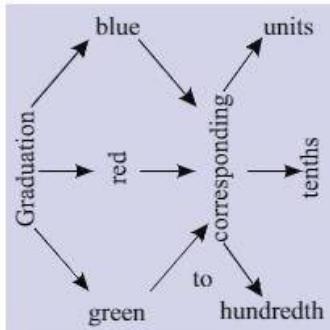
Observe the graduation given below:
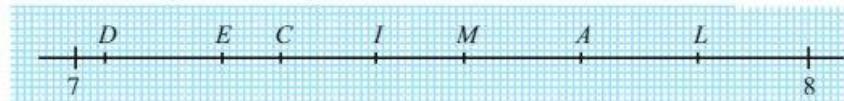
a) Write the decimal numbers corresponding to the points D; E; C; I; M; A and L.

b) Write each of these numbers in the form of:
- a decimal fraction.
- a sum of an integer and decimal fractions.

c) Copy and complete the table:

D				
E				
C	7.28	$\frac{728}{100}$	$7+\frac{2}{10}+\frac{8}{100}$	$(7\times1)+(2\times0.1)+(8\times0.01)$
I				
M				
A				
L				

Tenth
$\frac{1}{10}=0.1$

Thousandth
$\frac{1}{1000}=0.001$

Hundredth
$\frac{1}{100}=0.01$

I remember that!
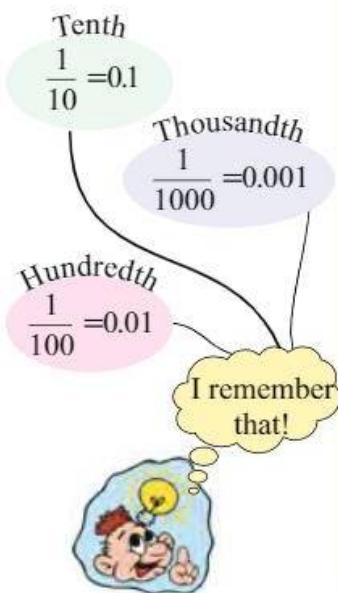
d) Write the following sums as decimal numbers:

$$7+\frac{4}{10}+\frac{8}{100} \qquad ; \qquad 25+\frac{1}{10}+\frac{6}{100}+\frac{9}{1000}$$

$$38+\frac{5}{100}+\frac{3}{1000} \qquad ; \qquad 60+\frac{5}{1000}+\frac{2}{10}$$

e) For each of the decimal numbers found, write its approximate value to the unit; to the tenth; to the hundredth.

5 units   5 parts of a thousand

1 5 3 5 . 5 2 5

5 hundreds   5 parts of ten

4   4.65   5

The approximation of 4.652

- **to the unit 4**
- to the tenth: 4.6
- to the hundredth: 4.65

Approximate = Cut

The rounding of 4.652

- **to the unit 5**
- to the tenth: 4.7
- to the hundredth: 4.65

• A decimal number can be written:
  - In digits: 236.79 .
  - In letters: two hundred thirty-six and seventy nine hundredths.

  - In form of:
    - a decimal fraction : $\dfrac{23\,679}{100}$
    - a sum of terms (decomposition)

    $$236 + \dfrac{7}{10} + \dfrac{9}{100} \qquad \text{or} \qquad 236 + \dfrac{79}{100}$$

    or $(2\times100)+(3\times10)+(6\times1)+(7\times0,1)+(9\times0,01)$.

  - In form of a mixed number: $236\,\dfrac{79}{100}$.

• Certain decompositions allow to determine the number of:

  - units $7.28 = 7 + \dfrac{28}{100}$ ⟶ **7 Units**

  - tenths $7.28 = \dfrac{72}{10} + \dfrac{8}{100}$ ⟶ **72 tenths**

  - hundredths $7.28 = \dfrac{728}{100}$ ⟶ **728 hundredths**

Approximation - Rounding:
The approximation to the unity of a number is its whole part.
Rounding to the unit number, is giving the whole number (integer) that is closest to it.

Approximation of a decimal number:

- to the tenth
- to the hundredth
- to the thousandth

is the decimal number having to the most

digit(s) after the point and we drop the other digits to the right

Rounding of a decimal number:

- to the tenth
- to the hundredth
- to the thousandth

is the decimal numbre having the most

digit(s) after the point and the closest to the given number

The number 53.829 is the quotient of the division of 53829 by 100.

$$53.829 = \frac{53\,829}{1\,000} = \frac{50\,000}{1\,000} + \frac{3000}{1000} + \frac{800}{1000} + \frac{20}{1000} + \frac{9}{1000}$$

$$= \; 50 \; + \; 3 \; + \; 0.8 \; + \; 0.02 \; + \; 0.009$$

$$= (5 \times 10) + (3 \times 1) + (8 \times 0.1) + (2 \times 0.01) + (9 \times 0.001)$$

$$= (5 \times 10) \; + (3 \times 1) \; + \; \frac{8}{10} \; + \; \frac{2}{100} \; + \; \frac{9}{1000}$$

fifty-three point height hundred twenty-nine

Whole part **decimal part**

fifty-three units and eight hundred twenty-nine thousandths

5 3 . 8 2 9

In this number 53.829 there are:

$53 + \dfrac{829}{1000}$

53 units

$\dfrac{538}{10} + \dfrac{29}{1000}$

538 tenths

$\dfrac{5382}{100} + \dfrac{9}{1000}$

5382 hundredths

$\dfrac{53829}{1000}$

53829 thousandths
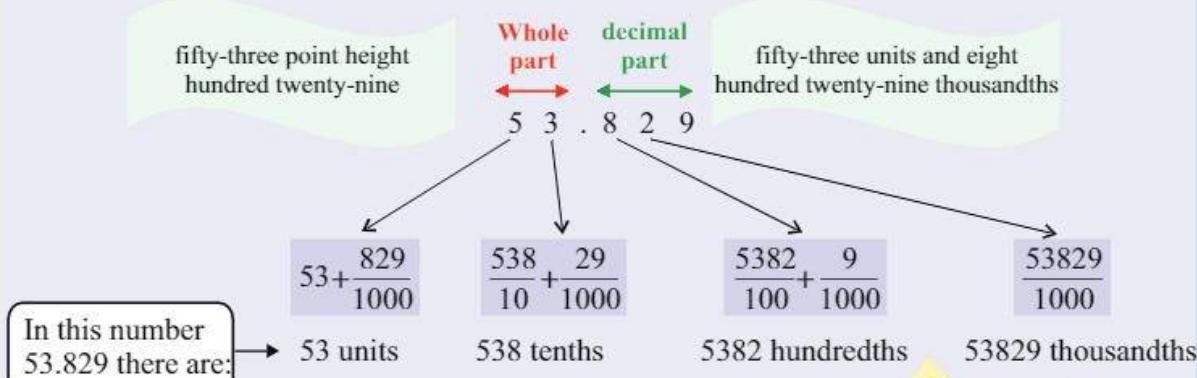
To "approximate" a decimal number, we "cut" at the range indicated and ignore (drop) the digits to the right.

To round a decimal number, we approximate first at the range indicated then:
- if the digit dropped is less than 5, we keep the approximation as it is.
- if the digit dropped is greater than 5, we add one to the last digit of the approximation.

	Approximation	36.528	Rounding	
to the unit	36	after the 6 we have "5"	37	to the unit
to the tenth	36.5	after the 5 we have "2"	36.5	to the tenth
to the hundredth	36.52	after the 2 we have "8"	36.53	to the hundredth

1- Write in digits the following numbers:
- Five thousand thirty.
- Forty-five units and eight hundredths.
- Thirteen units and nine thousandths.
- Five units and eighteen thousandths.
- Thirty three tenths.
- Seventy-five thousandths.

2- Write in letters the following numbers:
2 080 - 11.2 - 13.85 - 6.09 - 54.023 - 0.97 - 340.216 - 0.08.

3- Observe the example, then decompose in the same way the following numbers:

Ex: $45.27 = (4 \times 10) + (5 \times 1) + (2 \times 0.1) + (7 \times 0.01)$

$45.27 = (4 \times 10) + (5 \times 1) + \dfrac{2}{10} + \dfrac{7}{100}$

39.46 ; 461.23 ; 3009.57 ;
78.03 ; 40.104 ; 0.38 .

4- Write the decimal numbers corresponding to the following decompositions:

$(5 \times 10) + (6 \times 1) + \dfrac{7}{10}$; $(7 \times 1) + \dfrac{1}{100} + \dfrac{2}{1000}$

$(9 \times 1000) + (2 \times 1) + \dfrac{8}{100} + \dfrac{7}{1000}$;

$(4 \times 100) + (5 \times 1) + \dfrac{8}{10} + \dfrac{5}{1000}$; $\dfrac{4}{1000} + \dfrac{6}{10}$; $\dfrac{7}{10000}$.

5- Write the following numbers in the form of:
a) a decimal fraction.
b) The sum of a whole number and decimal fractions.
6.8 ; 29.67 ; 4.05 ; 49.135 ;
32.083 ; 17.002 ; 0.000 003.

6- Write in a decimal form the following numbers:

$5 + \dfrac{8}{10}$; $18 + \dfrac{7}{10} + \dfrac{5}{100}$; $\dfrac{9}{100\,000}$;

$24 + \dfrac{3}{10} + \dfrac{7}{100} + \dfrac{9}{1\,000}$; $9 + \dfrac{2}{1\,000}$;

$38 + \dfrac{1}{10} + \dfrac{5}{1\,000}$; $512 + \dfrac{1}{10} + \dfrac{6}{10\,000}$.

7- A math teacher dictated to his students the number "five-hundred-eight thousandths".
There were five different answers:
5 008 000 ; 508 000 ; 0.508 ; 0.00508 ; 5 008.000.
Which of these is the right answer?

8- Cancel the unwanted zeros, then write the following numbers as a decimal fraction:
340.170 - 08.205 - 002030.1080 - 0304.0060 .

9- From the following numbers, which have the same digit of tenths?

5.34 ; 18.46 ; $5 + \dfrac{4}{10}$; 48.75 ;

0.41 ; $8 + \dfrac{4}{100}$.

10- Put the point in the following numbers so that 5 becomes the digit of tenths:
7526 - 53 - 105 - 5 - 1658 - 5402.

11- Find among the following numbers those that have equal digits of hundreds and hundredths:
252.52 ; 639.69 ; 134.431 ;
821.084 ; 100.019 .

12- Copy and complete the following table:

Number	Decomposition	Numbers of tenths	Decomposition	Number of hundredths
6.352	$\dfrac{63}{10} + \dfrac{52}{1000}$	63	$\dfrac{635}{100} + \dfrac{2}{1000}$	635
48.139				
23.085				
14.607				

13- Copy and complete the following table:

| Number	digit of tenths	digit of hundredths	digit of thousandths
21.95			
7.14			
0.68			
0.435			
6.20			

14- Express the following lengths in *meters*, in *cm* and in *km*:
3*hm* 6*m* ; 5*dam* 2*m* 8*cm* ; 8*km* 7*dam* 6*dm* 9*dm* 3*mm* .

15- Express the following masses in *g*, in *mg*, then in *kg*:
5*hg* 8*g* ; 8*hg* 8*dag* 8*g* ; 9*kg* 6*g* 7*cg* ; 4*dg* 2*mg*.

16- Place the number 7.248 on the two following axes:
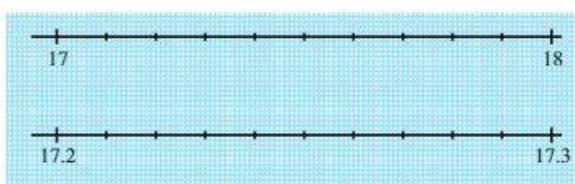
a) Is 17.248 closer to 17 or 8? to 17.2 or 17.3? to 17.24 or 17.25?

b) Copy and complete the following table:

	to the unit	to the tenth	to the hundredth
Approximation of 17.248			
Rounding of 17.248			

17- Which of the following numbers are equal to 34 when rounded to the unit?
3.41 ; 34.5 ; 34.17 ; 340.2 ;
$(3 \times 10)+(4 \times 1)+\dfrac{3}{10}$; $34+\dfrac{87}{1000}$.

18- Copy and complete the following tables:

Numbers \ Approximation	to the unit	to the tenth	to the hundredth
28.159			
6.093			
0.916			

Numbers \ Rounding	to the unit	to the tenth	to the hundredth
28.159			
6.093			
0.916			

What are the numbers that have the same approximation and rounding to the hundredth?

19- Give 5 different numbers that have 26.3 as approximation to the tenth and 26.4 as rounding to the tenth.

20- Give two numbers that have the number 29 for approximation to the unit and rounding to the unit.

21- Give a decimal number of equal approximation and rounding to the tenth but with different approximation and rounding to the hundredth.

22- Multiply 7436489.5 by 0.00001 .
a) Give the approximation to the tenth of the product.
b) Give the framing to the tenth of the product.

Self-evaluation

1- Copy and complete the following table:

Twenty-five units and thirty-eight hundredths	25.38	$\dfrac{2538}{100}$	$25 + \dfrac{38}{100}$	$25 + \dfrac{3}{10} + \dfrac{8}{100}$
	46.5			
		$\dfrac{45}{100}$		
			$3 + \dfrac{67}{1000}$	
				$8 + \dfrac{5}{100} + \dfrac{7}{1000}$
six hundred fifteen thousandths				

2- a) Complete the following table:

Number	15.085	42.543	0.801	24.706
App. to the unit				
Rounding to the unit				
App. to the tenth				
Rounding to the tenth				
App. to the hundredth				
Rounding to the hundredth				

b) What are the numbers that have equal approximation to the tenth and rounding to the tenth?

1- I am a decimal number with 2 digits behind the point. My digit of hundredth is 5. My number of tenth is 14. Who am I?

2- Imagine a riddle as in n°1 but for the number 5.813 and write it out.

3- Jad divides 2 squared pieces of paper into 100 small squares each.
a) On one of the squares, he colors a band in red and a little square in green.
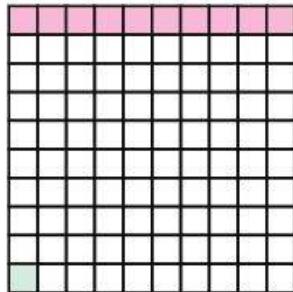
Complete the following statements:

- The band in red represents ..... of the square.

- The green small square represents ..... of the square

b) On a graph paper, draw 2 squares identical to Jad's.

On the 1st square, color	On the 2nd square, color
- in green, $\frac{7}{10}$ of the square	- in brown, $\frac{4}{10}$ of the square
- in yellow, $\frac{3}{100}$ of the square	- in pink, $\frac{6}{100}$ of the square

c) Jad does the following "pasting" on his notebook
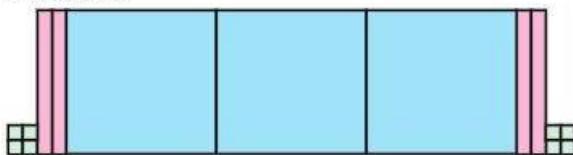
- How many whole squares does he use? Bands? Small squares?
- Represent your answer by one decimal number:
    "He used ... . Squares".

4- - The number 83 is closer to 80 than to 90, so the rounding of 83 to the tenth is 80, but the rounding to the tenth of 87 is 90.
a) What is the rounding to the tenth of:
    25 ; 42 ; 168 ; 53.7 ; 82.65 ?
b) Consider 4 car racing tracks of the following lengths: 32km - 27.5km - 35km - 24.8km. Calculate the total length of the race after rounding the length of each track:
    - to the tens of kilometers.
    - to the kilometer.

5- Rami has a wooden block that is1300 *mm*, long and Jihad has one that is 1400 *mm* long. They want to divide them into 6 blocks of equal lengths.
Calculate the length of each block rounded to the nearest *mm*.

6- At the supermarket, Ghada had the following receipt:

Supermarket			
Water	1	3389 LL	3.389
Laundry detergent	1	17604 LL	17.604
Detergent	1	2975 LL	2.975
Sugar	1	1350 LL	1.350
Labneh	1	2934 LL	2.934
Juice	3	578 LL	1.734
Soap	3	893 LL	2.679
Beans	1	1871 LL	1.871
Coffee	1	4366 LL	4.366
Lettuce	1	899 LL	0.899
TOTAL			

a) Using your calculator, find the exact total, then give its rounding to the thousandth.

b) Calculate the total by rounding the price of each item to the thousandth first.

c) Compare the two results.

You are on planet ERISTA. An interplanetary war has been declared. You want to go back to Earth in order to get help. Which way will you take?

There are dangers: the black holes, the uninhabited planets, and the enemies.

   - color in **black** the squares in which the numbers have for rounding to the unit 15.

   - color in blue those having for rounding to the tenth the number 28.3.

   - color in green those having for rounding to the hundredth the number 39.25.
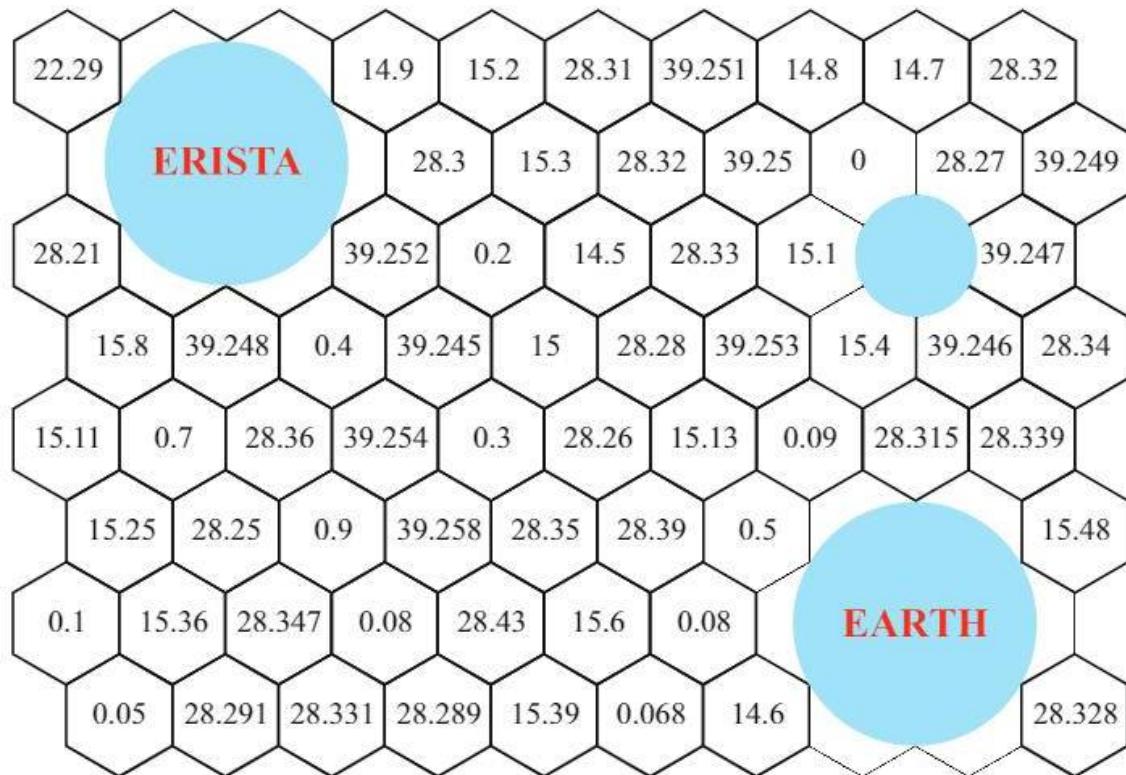
Central symmetry

Objectives

At the end of this Chapter, I will be able to:
- recognize and use central symmetry.
- find and characterize the symmetry elements of a plane figure.

Activities

A- Color the figure respecting a non-axial symmetry. Find the rule of the incomplete coloring and the colors of parts *a, b, c, d* and *e*.
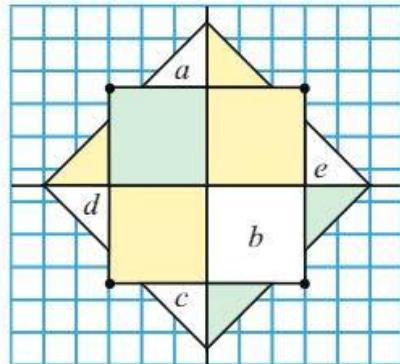
B- 1) Reproduce the drawing, given next, on a squared paper and draw the respective symmetrics:
- *A'* and *B'* of *A* and *B* with respect to (Δ).
- *C* and *D* of *A'* and *B'* with respect to (Δ').

2) Compare the lengths *OA* and *OC*, *OB* and *OD*, then the angles $A\widehat{O}C$ and $B\widehat{O}D$ to the straight angle. Determine the nature of quadrilateral *ABCD*.

3) Without referring to axial symmetrics, find a way to obtain the points *C* and *D* starting from *A* and *B*.

4) Construct segment [*EF*] of mid-point *O*.

**5)** Compare *AE* and *CF*, *BE* and *DF*, $A\widehat{E}B$ and $C\widehat{F}D$ and finally triangles *ABE* and *CFD*.
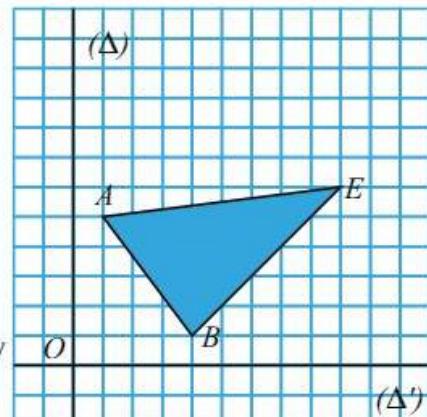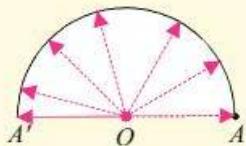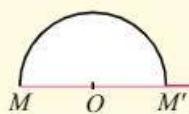
needle

Symmetric of a point with respect to a point O

● Consider two points O and A and a needle of length equal to OA.

One of the extremities of the needle is fixed at O, the other extremity touches A. Let us turn this needle around O.

When the needle has made a semi-round, the free extremity (mobile extremity) occupies the place of A' such that O is the mid-point of $[AA']$.

We say that:

A' is the symmetric of A with respect to O.

or:

A' is the image of A by the symmetry around O.

or even:

O is the midpoint of $[AA']$.

To obtain the symmetric M' of a point M with respect to O:

- We draw the semi-line $[MO)$ and the semi-circle of center O and radius OM that cut $[MO)$ in M'.
- M' will be the symmetric of M with respect O.

The symmetric of a point M with respect to a point O is the point diametrically opposite to M in the circle of center O and radius OM.

Symmetric of a segment with respect to a point O

A' and B' being the symmetric of A and B, respectively, with respect to a point O. O is the common midpoint to segments $[AA']$ and $[BB']$, the diagonals of quadrilateral $ABA'B'$.

Hence:

$ABA''B'$ is a parallelogram and $[AB]$ and $[A'B']$ are parallel and equal.

The symmetric of a segment, with respect to a point O, is a segment parallel to it and having same length.
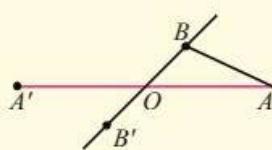
Symmetric of a figure

When drawing the symmetrics of the vertices of a figure (F) by symmetry around a point C, we obtain the vertices of figure (F') symmetric of (F) with respect to C.
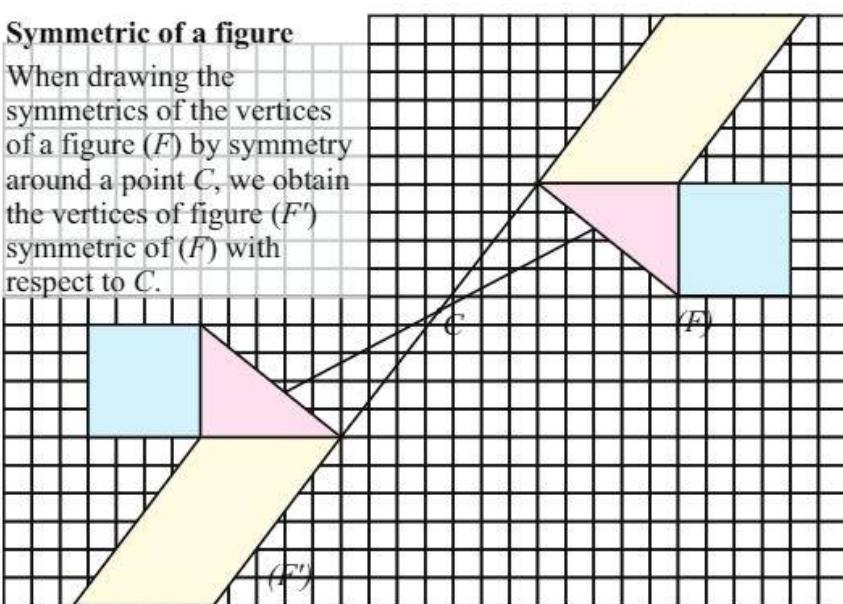

Properties of symmetry around a point

Let us compare the figures (F) and (F') where (F') is the image of (F) by symmetry around a point C.

(F) and (F') are suppressible, and the parts SYM, METQ and TRIQ of (F) are superposable respectively to the parts *sym*, *metq* and *tric*.
In other terms: The image of a triangle, a rectangle or a parallelogram is a figure identical to it and:
The symmetry around a point conserves the shape and dimensions of a given figure.
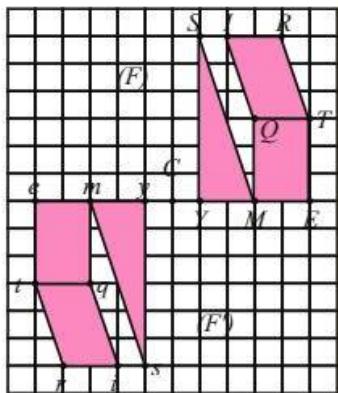
1) The point M is the mid-point of segment [YE], its image "m" is the mid-point of [ye] image of [YE].

2) The points S, I and R being collinear, their respective images s, i and r are also collinear.

3) The segments [SY] and [YE] are perpendicular and so are their images [sy] and [ye].

4) The sides [SM] and [QI] of the figure (F) are parallel, so are their images [sm] and [qi].
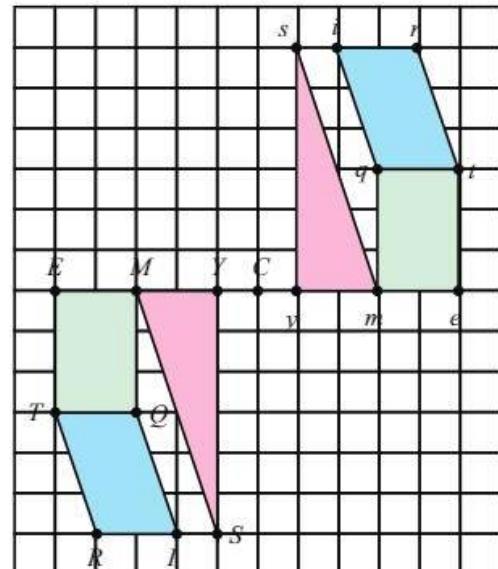
Therefore:
> The symmetry around a point conserves the midpoint, the linear, the right angle, and the parallelism.

89

Figures admitting a center of symmetry

$ABCD$ is a square and O is the point of intersection of its diagonals. Since O is the midpoint of $[AC]$, then the symmetric of vertex A with respect O is the vertex C. The same applies to B and D.

When applying the symmetry around point O, the image figure of square $ABCD$ will be the square itself. We say that O is a "center" of symmetry of the square.

When we are applying the symmetry around a point O, the image of a figure is itself, then we called the point O the center of symmetry of the figure.

**Examples**

From the following figures:
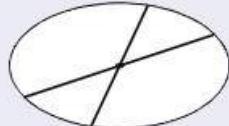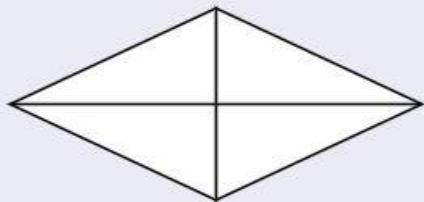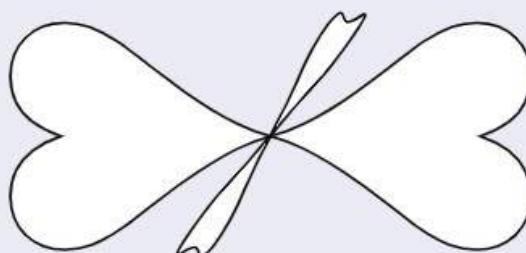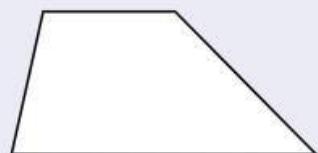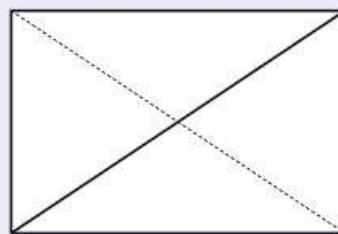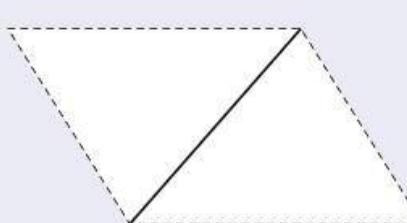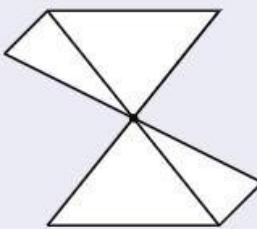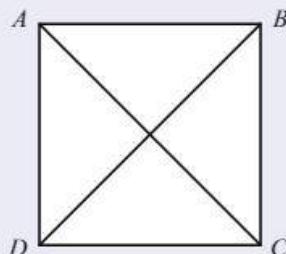
Only the trapezium does not have a center of symmetry.

Exercises

1- From the following figures select those admitting:
a) an axis of symmetry.
b) two axes of symmetry.
c) a center of symmetry.
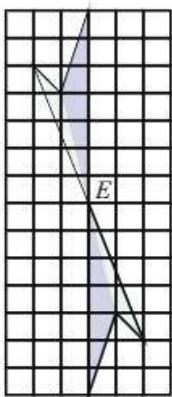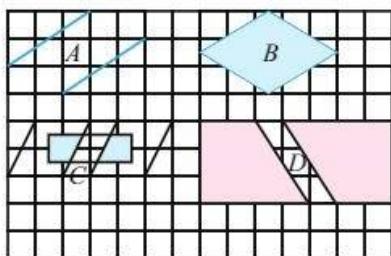
2- Reproduce each of the following figures and construct its symmetric with respect to a point.

a)

b)

c)
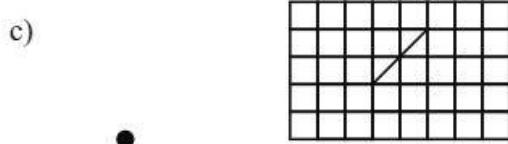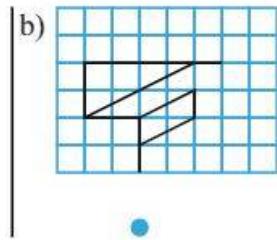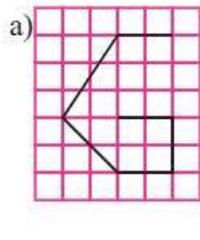
3- Reproduce and complete the figures below in a way that they have a center of symmetry.

a)

b)

c)
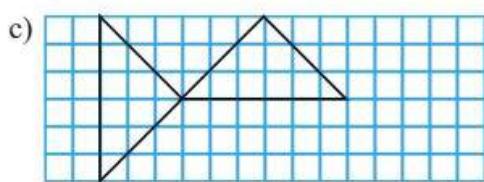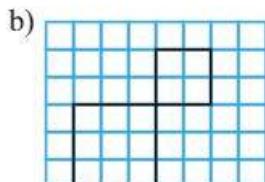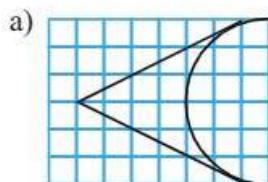
4- Can you draw a triangle having:
a) an axis of symmetry?
b) two axis of symmetry?
c) three axis of symmetry?
d) a center of symmetry?
Justify each answer using a drawing.

5- Draw a parallelogram and a circle that have the same center of symmetry.

6- Using only a ruler (nongraduated), find the center of symmetry for each of the following figures:

a)

b)

c)
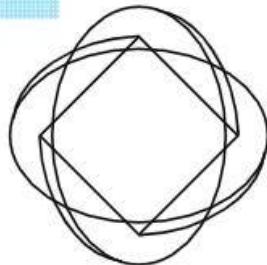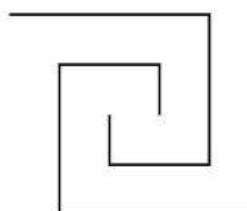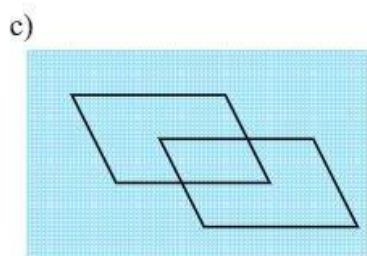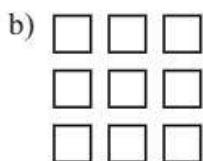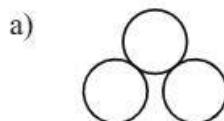
7- Complete the following figures in a way that they have a center of symmetry:

a)

b)

91
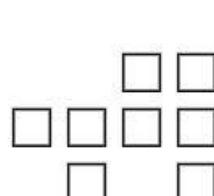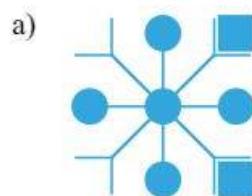
1. Complete the figure in a way that it has a center of symmetry.
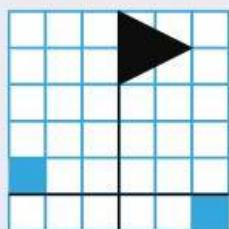
2. Reproduce the figure below, then construct its symmetric with respect to a point marked in blue.
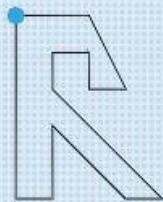
Problems

1- Draw a square *ABCD* of side 3*cm*.
a) Draw a rhombus *EFGH* that has a point *A* as center of symmetry.
b) Construct the image of the figure obtained by symmetry around point *G*.

2- Draw two parallelograms that have:
a) The same diagonal.
b) The same center of symmetry. What do you notice?
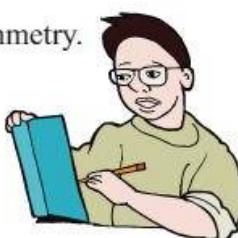
**3-** a) A rectangle and a parallelogram have the same center of symmetry. Construct this figure.
b) Construct a rectangle and a circle that have the same center of symmetry. Draw all possible cases.
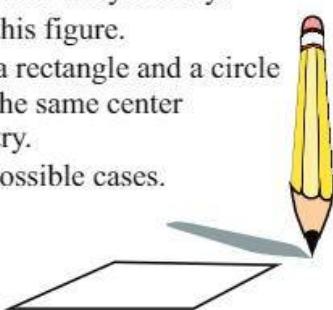
4- a) Determine the axis of symmetry, then the center of symmetry of the following figure:
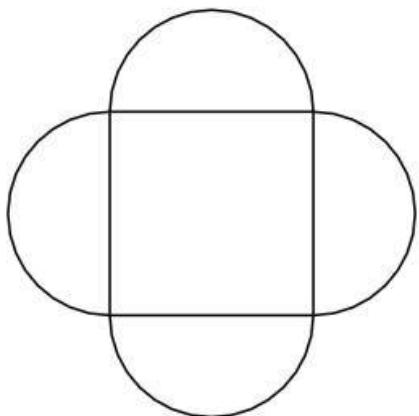
b) Using only a compass, find the center of symmetry of square *ABCD* (Explain while constructing).
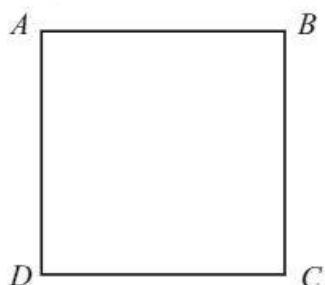
5- Observe the figure below:

a) Obtain the necessary measurements to calculate the perimeter and area of the rectangle and semidiscs.
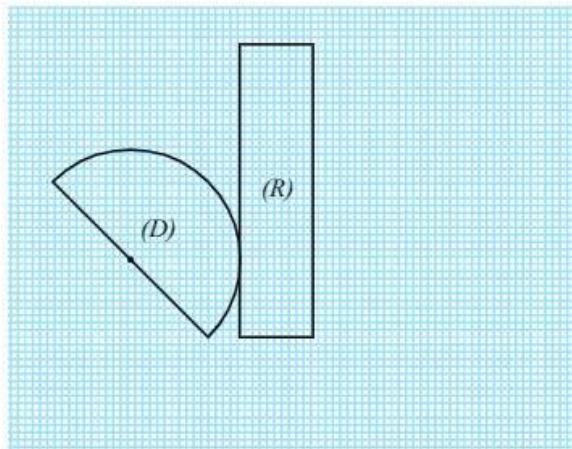
b) Reproduce the figure, then construct its symmetric with respect to point O.

c) Determine:
- the dimensions, the perimeter and the area of the rectangle (R'), image of (R) by this symmetry;

- the radius, the perimeter and the area of (D'), image of (D).

d) Compare the results obtained to those of a).

6- a) Determine the center of symmetry O of the figure (F) below:
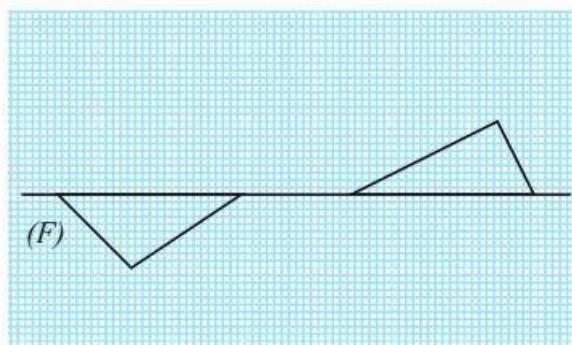
b) Locate on (F) a rectangle to obtain a figure (F') admitting O as center of symmetry.

c) Locate on (F') a circle in a way that the obtained figure has O as a center of symmetry.

7- a) Reproduce the figure $RAIE$ on a graph paper then determine respectively its images as:

- symmetry around O;
- symmetry of axis (Oy).
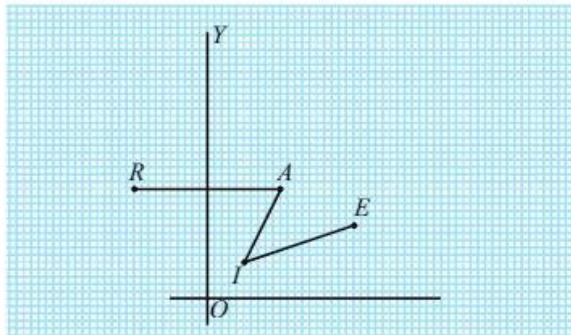
b) Starting from $RAIE$, find a way to obtain a figure that has a center and two axes of symmetry.

c) Find the center of symmetry of $RAIE$.

8- Copy the drawing a, b, c and d, each on a graph paper.
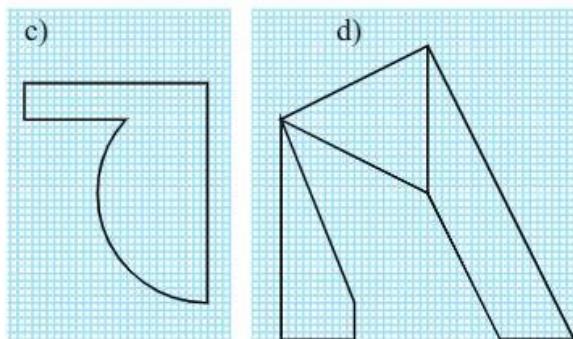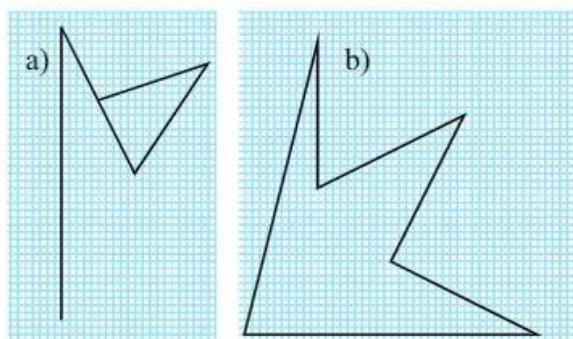
Construct the four figures F_1, F_2, F_3 and F_4 which have a center and two axes of symmetry, where $a)$, $b)$, $c)$ and $d)$ are respectively parts of them.

Copy and cut each of these figures; Use all cut pieces to make a figure that has a center of symmetry.

Calculating areas

15
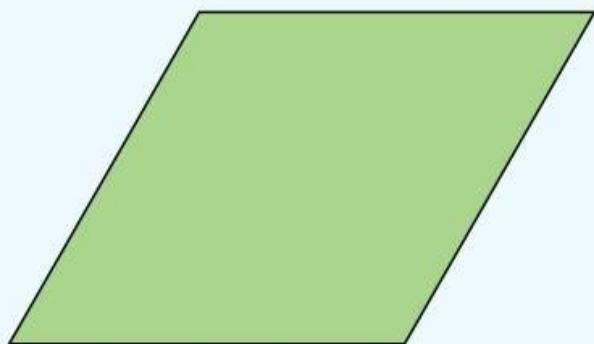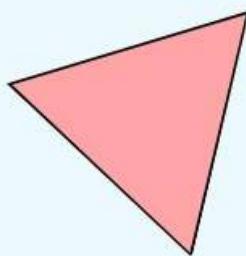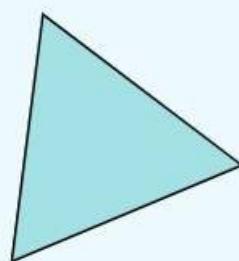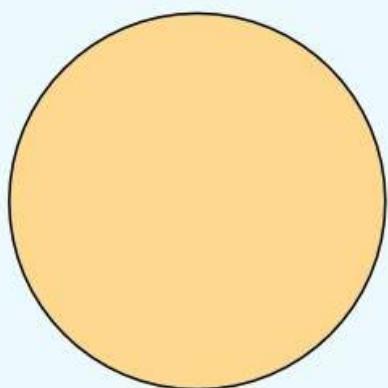
Objectives

At the end of this chapter, I will be able to:
- construct the metric system of unit areas.
- construct a unit of area: square meter, square centimeter, and square decimeter.
- establish the relations between the units of area.
- use the square decameter, square hectometer and square kilometer.
- correctly use the symbols of the metric units of area.
- perform conversions between different units of area.

Activities

1dm

$1mm^2$

$1cm^2$

There is 100 mm^2 in a cm^2.

Activity 1

a) On a 1 mm^2 graph paper, draw 2 squares; one with a side of 1cm and the other with a side of 1 dm.
How many small squares do we need to cover the big square?
If the area of the small square is 1 cm^2, then what is the area of the big square?
Calculate the area of a square whose side is 1 m.

b) Copy and complete:

$1cm = $ .... mm

$1cm^2 = $ ....mm^2

$1dm = $ .... cm

$1dm^2 = $ ....cm^2

$1m = $ .... dm

$1m^2 = $ ....dm^2

$1m = $ .... cm

$1m^2 = $ ....cm^2.

c) Find in cm^2 the area of each of the adjacent figures.

Activity 2

a) Draw a parallelogram *ABFD* and measure the length of its sides [*AB*] and [*DF*].
To find a rectangle from this parallelogram, follow the steps: I, II, III:

(I)   **(II)**   **(III)**
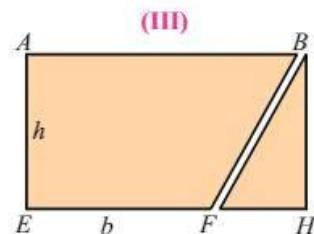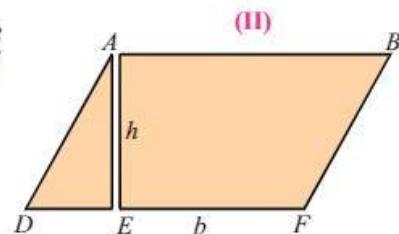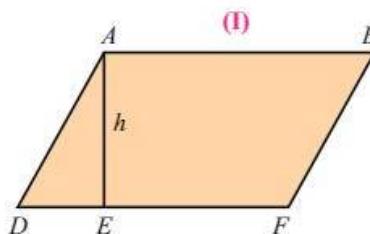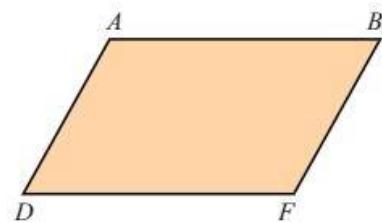
> "*h*" is the length of the height drawn perpendicular from *A* to the side [*FD*].

Calculate in *cm²* the area of the parallelogram *ABFD* and the area of the rectangle *ABDE*. Write the formula that enables you to calculate the area of a parallelogram.

b) Following the steps *a)*, *b)*, *c)* and d below, compare the area of the triangle *DEF* to the area of the rectangle *GHEF*:

> "*h*" is the measurement of the height drawn perpendicular from *D* to side [*FE*].

a)   b)

c)   d)
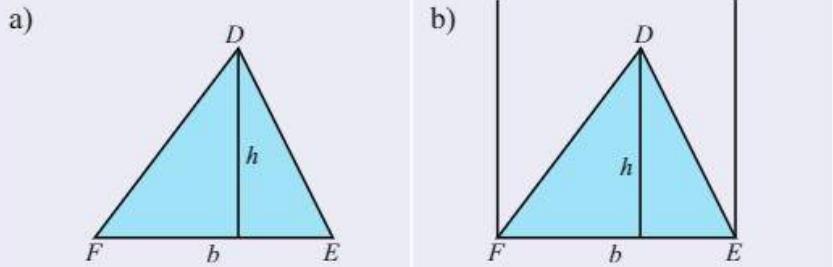
> "*b*" is the measure of the side's length [*FE*].
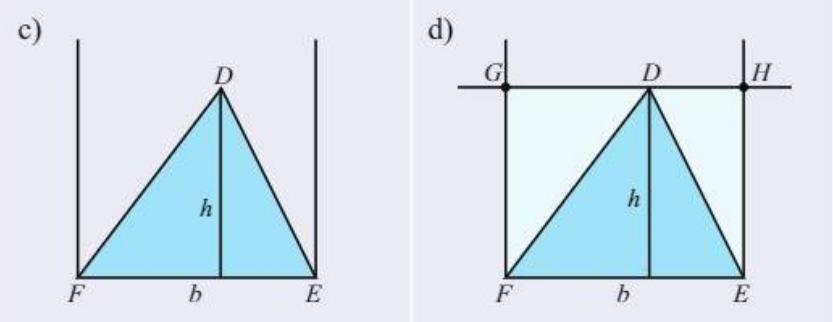
Write the formula that enables you to calculate the area of a triangle.

1 cm^2 is the area of a square of 1 cm side.

$1cm^2$

$are = 100\ m^2$

$hectare = 100\ are.$

- As the lengths are measured by the following units:
  $km, hm, dam, m, dm, cm, mm,$
  the areas are measured by the following units:
  $km^2, hm^2, dam^2, m^2, dm^2, cm^2, mm^2.$

- km^2 is the area of a square of 1 km side.

- We can also use the area and the hectare as units of area for land.
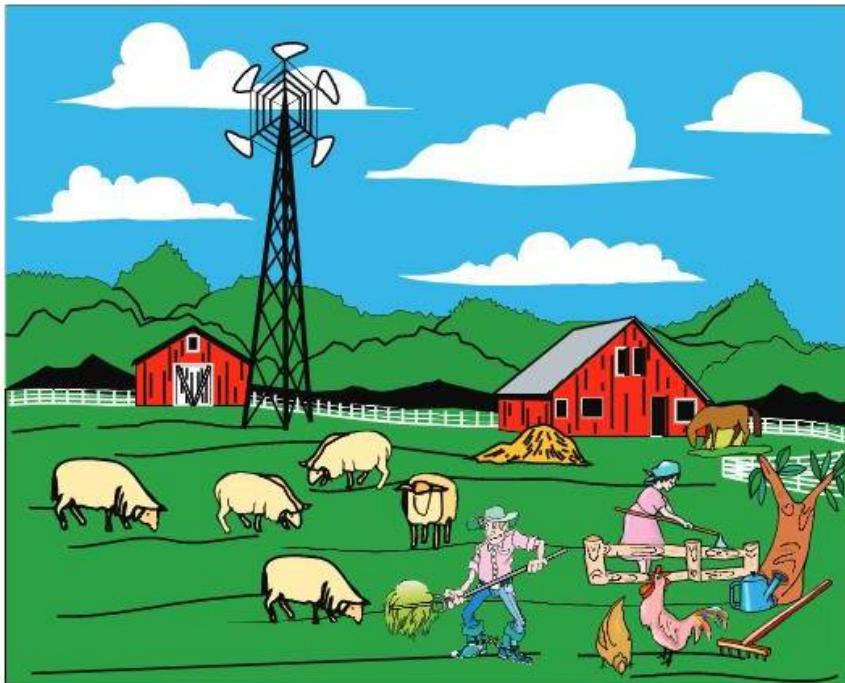
- **In order to calculate the area of a:**

  * rectangle:
  we calculate the product
  of its dimensions

  $\mathscr{A} = L \times w$

  $L = 4cm$

  $w = 2cm.$

  $area = length \times width$

  $\mathscr{A} = 4cm \times 2cm.$
  $\mathscr{A} = 8cm^2.$

  * square:
  we calculate the length of
  its side multiplied by itself.
  area =
  length of side $\times$ length of side

  $\mathscr{A} = S^2$

  $S = 2cm$

  $\mathscr{A} = 2cm \times 2cm$
  $\mathscr{A} = 4cm^2.$

* parallelogram:
Calculate the product of the length of its side and the corresponding height.

$h = 0.8cm$

$b = 3cm$
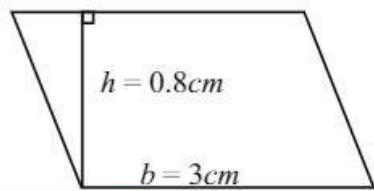
$\mathscr{A} = 3cm \times 0.8cm$
$\mathscr{A} = 2.4cm^2$

| Area = length of side $\times$ corresponding height |
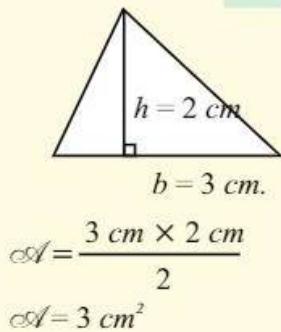
* triangle:
Calculate half the product of the length of the base and the corresponding height.

$h = 2\ cm$

$b = 3\ cm.$

$\mathscr{A} = \dfrac{3\ cm \times 2\ cm}{2}$

$\mathscr{A} = 3\ cm^2$

| $area = \dfrac{\text{Lenght of the base} \times \text{height}}{2}$ |

* disc:
Calculate the product of "π" and the square of the radius.

$r = 2cm$

| $area = \pi \times radius \times radius = \pi . r^2.$ |
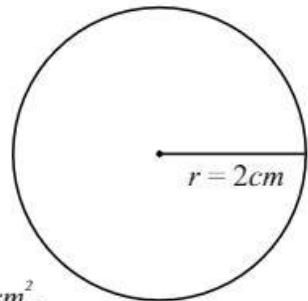
$\mathscr{A} = 3.14 \times 2cm \times 2cm = \mathscr{A} = 12.56cm^2.$

Focus

- The table below shows the units of area:

$\times 100$ $\times 100$ $\times 100$

km^2	hm^2	dam^2	m^2	dm^2	cm^2	mm^2
		7 0 0				

$:10^2$ $:10^2$ $:100$

$7hm^2 = 7 \times 100\ dam^2$
$7km^2 = 7 \times 1000\ dam^2$

- The area of a:

rectangle:	Square:	Parallelogram:	Disc:	triangle:
$\mathscr{A} = \ell \times w$	$\mathscr{A} = S \times S$	$\mathscr{A} = S \times H$	$\mathscr{A} = \pi \times r^2$	$\mathscr{A} = \dfrac{b \times h}{2}$
ℓ = length w = width a = area	$a = S^2$ S = side	S = side's length H = adjacent height	r = radius $\pi = 3.14$	b = base's length h = length of corresponding height
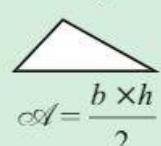
1- Convert the following:

$1\ km^2 = \ldots\ldots\ldots\ldots\ hm^2$

$1\ hm^2 = \ldots\ldots\ldots\ldots dcm^2$

$1\ dam^2 = \ldots\ldots\ldots\ldots m^2$

$1\ m^2 = \ldots\ldots\ldots\ldots dam^2$

$1\ dm^2 = \ldots\ldots\ldots\ldots cm^2$

$1\ cm^2 = \ldots\ldots\ldots\ldots mm^2$.

2- Copy and complete:

m^2	cm^2
67.9	.....
.....	8284
.....	48620
9	.....

.....

3- Convert the following:

$5.6\ hm^2 = \ldots\ldots\ldots\ldots m^2$

$136800\ mm^2 = \ldots\ldots\ldots\ m^2$

$0.25\ m^2 = \ldots\ldots\ldots\ldots cm^2$

$34\ dm^2 = \ldots\ldots\ldots\ldots dam^2$

$15.234\ km^2 = \ldots\ldots\ldots\ m^2$.

4- Convert to are:

$5\ ha$; $17.3\ ha$; $100\ m^2$;

$2.4\ m^2$; $3.2\ dam^2$; $62\ hm^2$.

5- Copy and complete:

$2\ m^2 + 3\ dam^2 = \ldots\ldots\ldots\ldots m^2$

$17.5\ cm^2 + 0.35\ m^2 = \ldots\ldots\ldots mm^2$

$73.9\ are - 17\ m^2 = \ldots\ldots\ldots\ldots m^2$

$9378\ mm^2 - 25\ cm^2 = \ldots\ldots\ldots cm^2$

$2\ ha + 17\ are + 23\ m^2 = \ldots. m^2$

$0.8 dm^2 + 72438 mm^2 + 95.5 cm^2 = \ldots cm^2$

$2.5 m^2 + 0.8 dm^2 - 10.2\ cm^2 = \ldots\ldots mm^2$

$0.03\ hm^2 + 0.5\ m^2 - 17 dm^2 = \ldots. cm^2$.

6- Copy and complete the following table:

rectangles	length's measure	width's measure	area
R_1	25cm	13cm	
R_2	2hm	45dm	
R_3	34cm		$734.4 cm^2$
R_4		38dm	$24.7 m^2$

7- Choose the right answer:

$1\ are = 1\ m^2$; $1\ dm^2$; $1\ dam^2$.

hectare $= 1\ km^2$; $1\ hm^2$; $1\ dam^2$.

8- Copy and complete the following table:

Squares	Side measurements	Area
S_1	32cm	
S_2	7.5km	
S_3		$25 dm^2$
S_4	1.2m	
S_5		$144 cm^2$

9- Calculate the area of the following figures:

8 dm

21 dm

8 cm

16 cm

3 dm

4,85 m
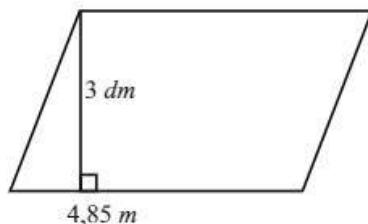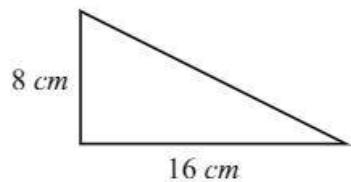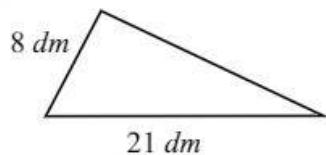
10- Copy and complete the following table:

Discs	Radius	area
D_1	25.3cm	
D_2	192dm	
D_3		$78.5m^2$
D_4		$314dm^2$
D_5	6m	

11- a) The area of the parallelogram $ABCD$ is $182\ cm^2$, the length of its side $[AB]$ is $36\ cm$

.   Calculate the length of the height relative to side $[AB]$ of $ABCD$.

b) The area of the triangle ADF is $34\ dm^2$, $DF = 142\ cm$. Calculate the length of the height of this triangle relative to $[DF]$.

Self-evaluation

1) Calculate in cm^2 the area of a parallelogram given that the length of its sides is = 8.5cm and the corresponding height is 5.5 cm.
2) Calculate the height corresponding to $[LM]$, in triangle KLM of area $65.2\ cm^2$ and its side $[LM]$ is 8.4 cm.

Problems

1- What are the appropriate units used to calculate the area of each of the following:
- house.
- play ground.
- field.
- country.
- copy book paper.
- 500 LL coin.

2- Find in each of the following cases the most realistic answer:

- Area of the class room:
  $81\ dm^2$; $900\ cm^2$; $50\ m^2$.

- Area of the paper in a math book:
  $5\ dm^2$; $25\ cm^2$; $10\ 000\ mm^2$.

- Area of Lebanon:
  $10\ 452\ m^2$; $10\ 452\ km^2$; $10\ 452\ are$.

- Area of a house:
  $7\ km^2$; $160\ m^2$; $1\ 000\ dm^2$.

3- Calculate in mm^2 the area of the figure below:
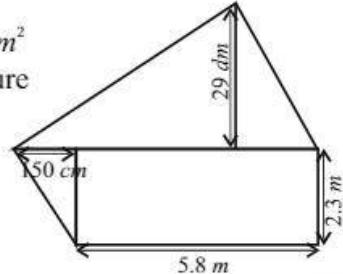
4- Calculate the area of the orchard represented in the figure below:
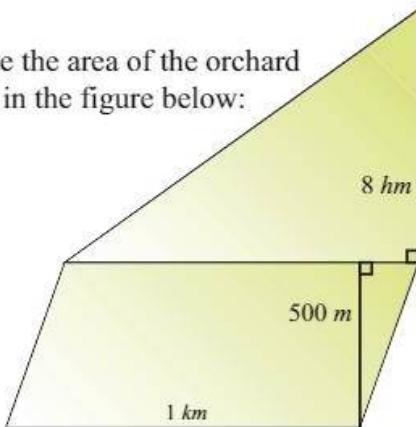
What is the selling price of this orchard if each m^2 is sold for 85 000 LL?

5-

24 m

100 m

R=15 m

r=12 m
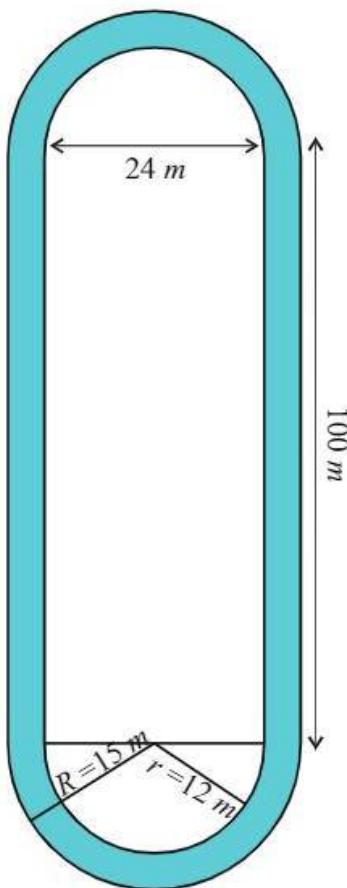
Calculate the colored area?

6- We planted potatoes in a triangular field that has a base whose length is 86 *m* and height 112 *m*.

What should the crop's weight be if the production of 1 *are* of field is estimated at 200 *kgs* of potatoes?

7- The owner of a land shaped as a parallelogram of 125 *m* of side and 22.8 *m* of relative height has applied for a license to build a rectangular house of 12.5 *m* and 9 *m* in dimensions, with a square shaped car garage 6 *m* per side.

A- Calculate the area needed for the house and the car parking.

B- Will the owner get the license knowing that, by law, the construction area shouldn't exceed 1/3 of the total area of land?

8- A triangular table is covered with a glass sheet which costs 19.500 LL per m^2.

60 cm

60 cm
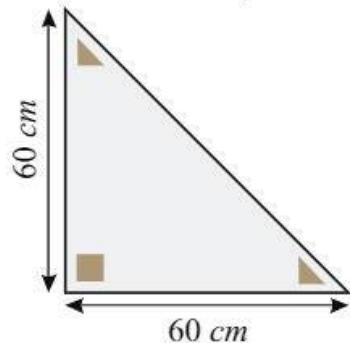
What is the price of the sheet?

9- To decorate a circular table of 50 *cm* in diameter, the woman, knitted a circular cloth with a diameter smaller than the table's by 15 *cm*.
What is the area of the cloth?

10- Find in *cm* the area of the colored part in the figure below.

2 cm   5 cm   2 cm

2 cm   2 cm
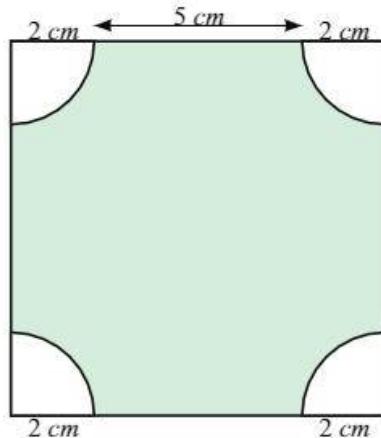
11- The base of a triangular boat is 2.20 *m* and its height is 3.50 *m*.
What is the price of the boat if it costs 32.850 LL/m^2?

12- The area of a flower-bed shaped right angled triangle is 20 m^2.One of the right angle's sides measures 4 *m*.
What is the other side's measure?

13- The area of a triangular half is 1394 m^2 and its height is 82 *m*.
What is the measure of the base?

14- After the edges of a rectangular carpet with the dimensions of 3 *m* and 2.4 *m* were worn out, we cut a 10 *cm* piece of the length and a 15 *cm* piece of the width.

Find the dimensions of the carpet; perimeter; area.

15- Find in m^2 the colored areas in each of the following figures below:

36 *m*

30 *cm*

50 *cm*

30 *cm*
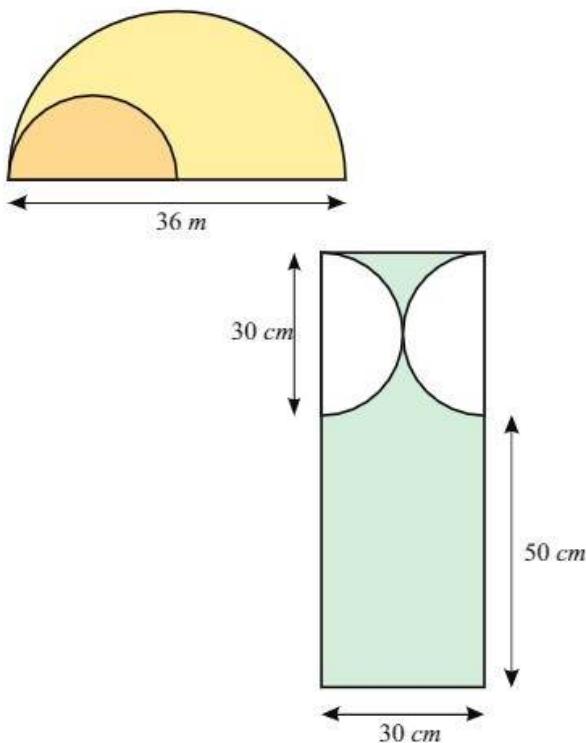
16- The dimensions of a party hall are: Length: 14.5 *m* ; Width: 9.7*m* ; Height: 3.6*m*.

The cost of painting 1 m^2 of the ceiling is 15.000 LL and 1 m^2 of the walls is 12.500 LL. The hall has 6 windows and a door.

The area of each window is 2.8 m^2, and the area of the door is double the area of window. Calculate the total cost for painting this hall.

17- The drawing below represents a piece of land.

38 *m*

12 *m*

??
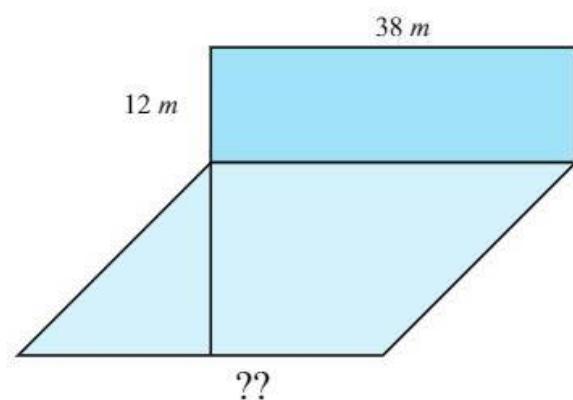
Calculate the height of the parallelogram in the drawing knowing that the land's total area is $1122\,m^2$.

Just for fun

Find the area of the colored part in the following figure.

5 *m*
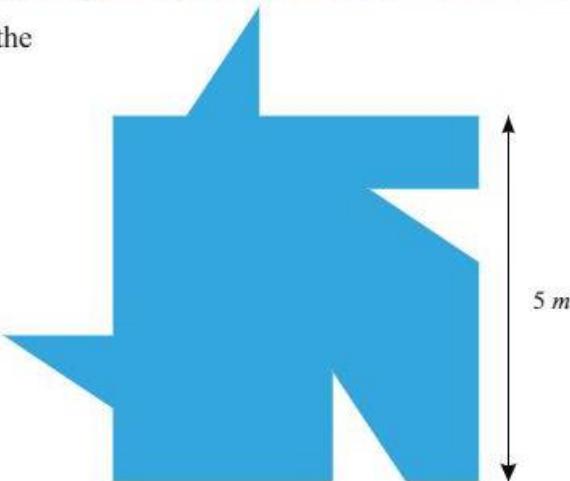
Calculation on literal expressions

16

Objectives

At the end of this chapter, I will be able to:

- ☐ Write formulas by using letters to replace known sizes;
- ☐ Use the distributivity of multiplication over addition in literal expressions;
- ☐ Calculate the numerical value of a literal expression.

Activities

Grade VI

b

a   *c*
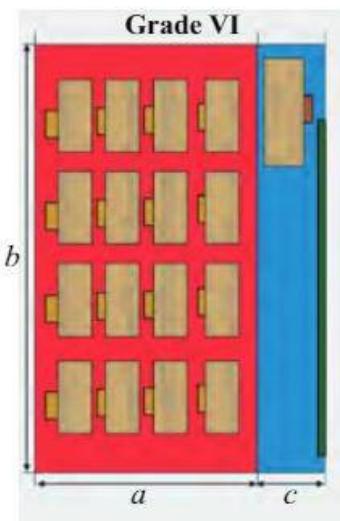
⌢ ⌢ ⌢ ⌢ ⌢
Remember
that the perimeter of a rectangle is the (length, plus the width) × 2

⌢ ⌢ ⌢ ⌢ ⌢
Remember
that the area of a rectangle Is the length times the width

A) This drawing shows the floor plan of a Grade VI classroom which has been furnished with one table and several chairs.

- Write a numerical expression showing the number of chairs in Grade VI.

- The number of Grade VI students is not known. Let m be the number of students.
  Write an expression to show how many chairs will be empty.

- In this class the number of girls is 12.
  Write an expression using m to show the number of boys in the class.

- To go on a trip, each student should pay 7500 LL.
  Write an expression to show the total money obtained if m students participated.

B) The chairs are put on a red rug and the table on a blue one.

- What is the perimeter of the blue and the red rug?
  Find using two different ways the area of the covered floor?

- •• What is the area of the blue and the red rug?
  Using two different ways find, the area of the covered floor.

- ••• Calculate $a \times (b + c)$ and $a \times b + a \times c$
  when $a = 3$; $b = 6$; and $c = 2$;

2x + 7

2x + 3y

Algebric expression

7 + (x-3)

$\frac{x}{4}$ +2

2 + (x-3)
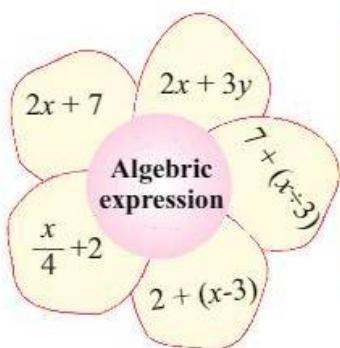
Expressions which include numbers and letters indicating operations are called algebric expressions.

These letters stand for numbers. If the numbers are known, the value of each expression can be found.

"Word Expression"

x more than 5.	x minus 9	x times 2	x divided by 7
The sum of x and 5.	x substracted from 9	x multiplied by 2	x over 7
the total of x and 5	9 less than x	double of x	A number x is distributed among 7
x increased by 5	x decreased by 9	twice of x	A rectangle of area
x plus 5	x daminshed by 9	the product to x and 2	x and one of its sides is 7, the other is $x \div 7$
5 years from now			

Algebric Expression

$x + 5$	$x - 9$	$2x$	$x \div 2$

Example 1:

$2x+23$ is an algebric expression for the word 23 more than twice of a number.

Example 2:

Twice the sum of a number and 23. Let x be the represented number, the algebric expression for this word is: $2(x + 23)$.

1- Twice a number more than triple another is: **word sentence between two phrases.**

2- Translate it to an algebric expression: $2x + 3y$

3- When $x = 2$ and $y = 3$. Replace the letters by their value: $2(2) + 3(3)$

4- Calculate the numerical value of the algebraic expression: $4 + 9 = 13$

Exercises

1- Write as an algebric (literal) expression:
- 13 more than x.
- x less than 15.
- The sum of x and y.
- 5 more than twice x.
- 7 added to triple Z.
- The product of 3 and c.
- x distributed among 7.
- x over double of 7.
- 7 greater than half of y.
- 21 more than quarter y.
- Z divided by 5.
- A number x increased by 3 times of y.
- A number x decreased by half y
- The area of a rectangle whose length is L and width is w.
- The area of a rectangle whose width is w and length triple the width.
- The perimeter of a rectangle whose width is w, and length is $5cm$ more than twice its width.
- The difference between triple x and its reciprocal.
- x multiplied by itself all divided by 6.

2- Copy and complete:

a)

a	b	c	$a+b$	$(a+b)+c$	$b+c$	$a+(b+c)$
2	5	7				
1.2	20	3.5				
3	7	2.3				
$a + (b + c)$ .... $(a+b)+c$						

b)

a	b	c	$a-b$	$(a-b)-c$	$b+c$	$a-(b+c)$
31	17	11				
14.2	6.5	0.8				
22	13	9				
$a - b - c$ .... $a - (b + c)$						

c)

a	b	c	$a \times b$	$(a \times b) \times c$	$b \times c$	$a \times (b \times c)$
9	5	7				
2.5	0.1	6.4				
12	5.3	2.7				
$(a \times b) \times c$ .... $a \times (b \times c)$						

d)

a	b	c	$b+c$	$a \times (b+c)$	$a \times b$	$a \times c$	$a \times (b \times c)$
4	3	7					
1.1	3.5	2					
3	2	2.7					
$a \times (b + c)$ .... $(a \times b) + (a \times c)$							

3- Write an expression to answer each question:

a- What is the product of x and 3.5?

b- What is one ninth of x?

c- What is the quotient of x and 42?

d- What is the difference between x and 11.4?

4- Complete the table by evaluating each expression for $x = 2$. 3 and 5

x	$x+43$	$30-x$	$x \times x$	$x \times 10$	$2x$	$\dfrac{150}{x}$	x^3
2							
3							
5							

5- Zeina brushes her teeth 3 times daily

Number of days	Number of brushing
1	3
4	
x	
$5x$	
$5x + 3$	

6- The product of two numbers is 32. If one of them is x, what is the other number?

Attention: x being not equal zero.

7- A dog has 4 legs.
How many legs do x dogs have?

8- The sum of four numbers is 100. Three of the numbers are 41, 9 and 25. What is the fourth number?

9- Let w be Abeer's weight last year. She followed a diet this year, and lost $13kg$. Write an expression for Abeer's weight this year.

10- Let d be the distance between stations A and B.
Rami ran a quarter of this distance. Write an expression for the distance Rami ran.

11- When 14 is added to double a number the result is 30. Find this number.

12- Farha is 11 years older than twice Marha's age.
Write an algebric expression for Farha's age.

13- Hiba is $7cm$ taller than Jana. Write an algebric expression to say how tall is Hiba.

14- There are x students in grade 6;
$\dfrac{3}{2}$ of the students are boys. Write an expression to show how many girls there are in grade 6.

15- A jar holds x kg of jam. How many jars are needed for 200 kg of jam?

16- Find the value of the following angles in terms of a:

a) $z\widehat{O}t$ b) $t\widehat{O}y$ c) $y\widehat{O}u$.
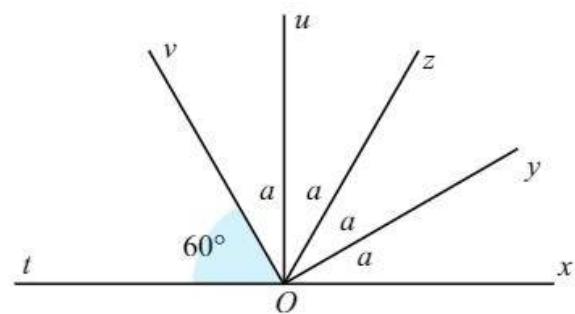
17- The sum of a certain number and its quarter is 20. Translate these words into an algebric expression.

18- When a certain number is multiplied by 8 the product is 136.
Translate into an algebric expression.

1- Answer by True or False:

a) $7x + 5 = 26$ when $x = 3$.

b) $2(x + 10) = 30$ when $x = 3$.

c) 17 *cm* taller than x is $(x - 17)$.

d) 6 more than double 7 all divided by 2 is: $2 \times 7 + 6 \div 2$

2- Copy and complete:

a	b	c	$a+(b+c)$	$a \times b \times c$	$a \times (b+c)$	$a+(b-c)$	$(a \times b)+(a \times c)$
21	11	5					
9.6	7.4	2					
3	2.5	1					

Problems

1-
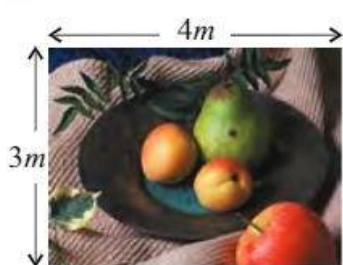
A   B

a) What is the area of picture *A*.

b) What is the area of picture *B*.

c) What is the length of the picture frame needed for *A* and for *B*?

d) What is be the cost of the picture frame if each *m* costs 250 LL?

2- Dina has 19 000 LL and Reem has 35 000 LL. How much would Reem give Dina in order for each to have the same amount?

3- The height of a right angle triangle is 3 times the length of the base. Find the area of this triangle.

4- The area of a square garden is 36 m^2. What is its perimeter?

5- Cut a piece of cardboard which is (a)cm long into 2 equal parts, then cut each part into 2 equal parts.

How long is each piece? Three pieces together?

6- Here is a plan for Mr. Nader's garden.
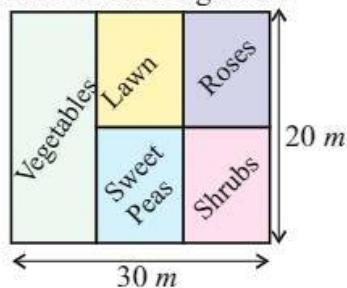
a) What is the area of this garden?
b) What is the area of the vegetable garden?
c) If the sweet peas were surrounded by a red fence and the roses surrounded by a green one, how many meters of fence would we need?
d) If each meter costs 700 LL, what is the cost of the fence?

Just for fun

Find the member

See if you can find this number. When you add eight to it, then subtract eight from the sum, then multiply the remainder by eight, then divide the product by eight, you get four.

What is the number?

Signed numbers

Objectives

At the end of this chapter, I will be able to:
- identify positive and negative signed numbers.
- locate signed numbers on a numerical axis.
- identify two opposite numbers.

Activities

a) Observe the following figure:

The yacht is at water level zero is the corresponding signed number.
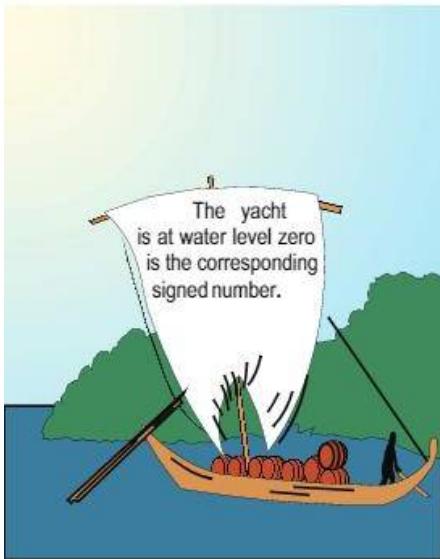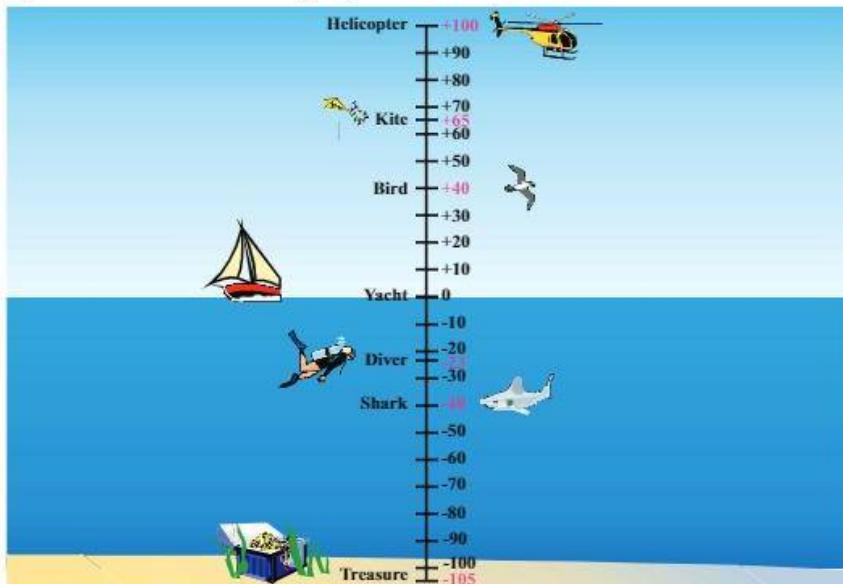
b) Copy and complete the following table:

Object	Position	Corresponding, signed number
Helicopter	100 *m* above water level	+100
Kite		
Bird		
Yacht		
Diver		
Shark	40 *m* below water level	-40
Treasure		

$+6.3$

zéro
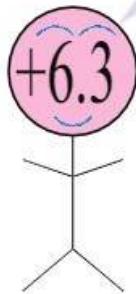
c) What characterizes the two numbers: +65 and -23, and what do they signify?

d) Reproduce the following axis:
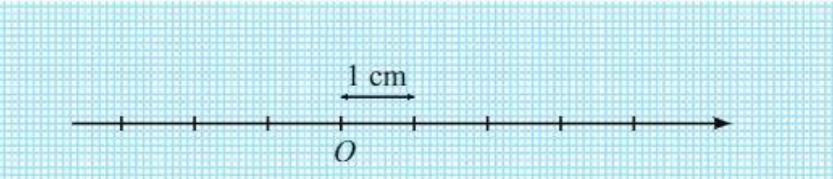
and locate the points H; K; B; Y; D; S and T corresponding to the numbers:

+100 ; +65 ; +40 ; 0 ; -23 ; -40 ; -106 .

(1 cm on the axis corresponds to 10 m in reality).

• What letter corresponds to zero?

• Copy and complete:

The numbers corresponding to the points located

• to the right of Y have the sign ....

• to the left of Y have the sign ....

e) Study the positions of the bird and the shark with respect to the water level. Then specify the position of Y with respect to $[BS]$.

Put the zero on the following axis:
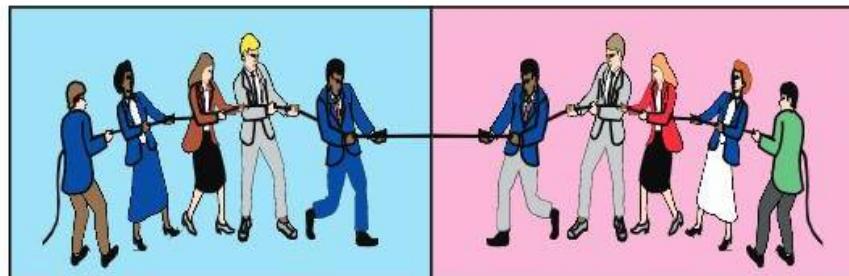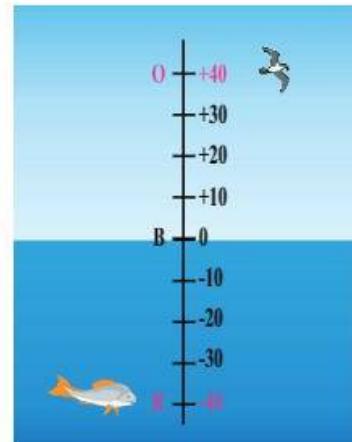
It is very hot. The temperature is +30°C

It is very cold. The temperature is -8°C

Sign   numerical part

| − | 25 |

signed number

```
     C    B         O              A    P
  ───┼────┼────┼────┼────┼────┼────┼────┼───►
    -3   -2         0   +1   +2   +3   +4
```
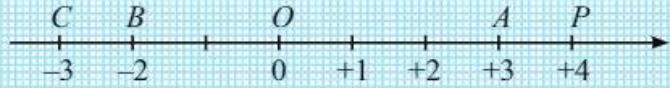
A number that depends on its sign is a signed number. If it is preceded by a sign :
+, then the number is positive.
-, then the number is negative.

Zero is the abscissa of O, where O is the origin of the axis.
+3 is the abscissa of A, the distance from A to the origin is 3.
-2 is the abscissa of B, the distance from B to the origin is 2.

The points A and C are symmetric with respect to the origin O: their abscissas are said to be opposite.
-3 and +3 are two opposite numbers
O is the midpoint of $[AC]$.

> Two numbers are said to be opposite if they are the abscissas of two points situated at equal distance from the origin.

Focus

```
    N    M    L    K    J    I    H    O    A    B    C    D    E    F    G
 ───┼────┼────┼────┼────┼────┼────┼────┼────┼────┼────┼────┼────┼────┼────┼──►
   -8   -7   -6   -5   -4   -3   -2   -1    0   +1   +2   +3   +4   +5   +6   +7
```
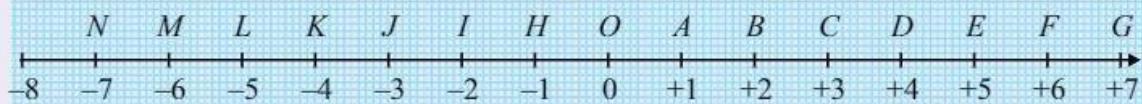
.............; -7; -6; -5; -4; -3; -2; -1 ; **0** ; +1; +2; +3; +4; +5; +6; +7; ....

The negative numbers   at the same time positive and negative   The positive numbers

signed numbers

B and K are two points symmetric with respect to the origin.
$+4 = \text{opp}(-4)$; $-4 = \text{opp}(+4)$; $OD = OK$.

Exercises

1- From the following numbers, choose those that are negative and those that are positive:

$+3.5;$ $-5;$ $+2.1;$ $-3;$ $+\dfrac{2}{5};$

$+\dfrac{3}{10};$ $+5.4;$ $0;$ $-2.5;$ $-3\dfrac{1}{4}.$

$+7\dfrac{1}{2}$ $-\dfrac{5}{4};$

Place them on a numerical axis.

2- Reproduce the following numerical axis:
What are the abscissas of the points:
$O;$ $E;$ $S;$ $A;$ $F;$ R and G?
Locate on the axis the points B and T of abscissas $-\dfrac{5}{2}$ and $+\dfrac{6}{10}$.
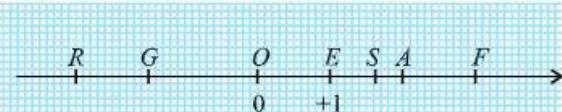
3- On a numerical axis, locate the points A; B; C; D; E and F of abscissas:
$-5;$ $+3;$ $-2;$ $+4;$ $+2$ and $+6$ respectively.
What are the abscissas of M midpoint of $[AB]$, and N the midpoint of $[CE]$?

4- Among the following numbers:
$+3.4;$ $-5.2;$ $-7.3;$ $+2.4;$ $+7.3;$
$+5.2;$ $-3.4;$ $-4;$ $-3.1;$ -2.5 ,
Indicate those that are opposites.

5- Find in each case the opposite of x (denoted by opp (x)):

a) $x = +3.6;$ d) $x = +5\dfrac{1}{4}$

b) $x = 0;$ e) $x = -\dfrac{5}{2};$

c) $x = -0.35;$ f) $x = +2.$

6- Copy and complete the following table:

a	$+6$		$+4.5$		$+6\dfrac{1}{2}$	
opp(a)		-3.4	$+2.1$		$-\dfrac{3}{2}$		$-3\dfrac{1}{4}$
opp(opp(a))							

Compare the first and the third lines.

+2 and -2 are two opposite numbers

Self-evaluation

1- a) Draw a numerical axis of origin O, of unit 1 *cm*, and locate the points P; L; U and S of abscissas: $+1;$ $+2;$ $+3$ and $+4$ respectively.
   b) Locate the points M, R, I and N the symmetric of P, L, U and S respectively relative to the origin, then find their abscissas.

2- Copy and complete the following table:

a	opp(a)	distance from a to the origin	distance from opp(a) to the origin
$+5$			
	-3.2		

1- Observe the following numerical axis:
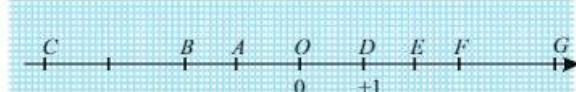
What are the abscissas of the points: A; B; C; D; E; F and G?

What are the pairs of points situated at an equal distance from O?

What can you say about such pairs? What can you tell about their abscissas?

2- Here are the grades over 20 of grade six:

Arabic Reading	12
Arabic Comprehension	9
Arabic grammar	14
English Reading	11
English Comprehension	$14\frac{1}{2}$
Mathematics	$15\frac{1}{2}$
Science	13
History-geography	8
Sport	15

Express using a signed number the spacing between each grade and 10.

3- a) In the first trimester, Rana obtained the following grades out of 20 in Mathematics:
14 ; 6 ; 8 ; 12 ; 13 .
Express using a signed number how far is each grade from 10.

b) In the second semester, Rana noted the position of her grades with respect to 10:
+6; +2; -3; +5; -1.
Find Rana's grades.

4- Draw a numerical axis of unit 1 cm, then locate these points on it:

$A(-4)$; $B(-2)$; $C(-1)$; $D(0)$; $E(+2)$;

$F\left(+\dfrac{3}{4}\right)$;

$H\left(+3\dfrac{1}{2}\right)$;

$G\left(+\dfrac{4}{5}\right)$;

$I\left(-1\dfrac{1}{4}\right)$.

$A(-4)$ is read: -4 is the abscissa of the point A.
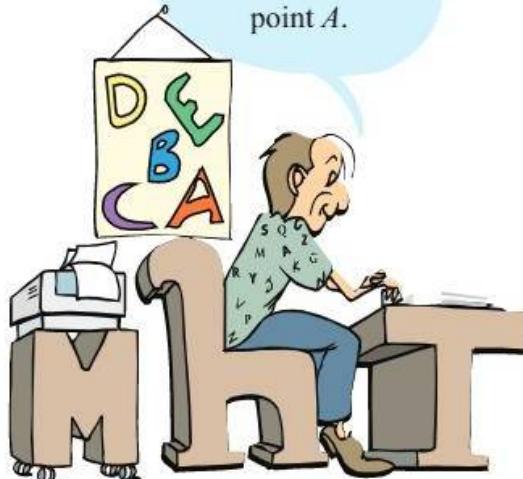
- What is the origin of this axis?
- Find the symmetric points with respect to the origin.
- Find the distance from each point on this axis to the origin.

Start from *A*, a vowel, advance by 4 letters, following the direction and the sense of the arrow, you will reach the letter *E*, also a vowel. Continue in the same way but: "if the letter reached is a vowel, advance by 4 more letters (+4)" and "if the letter reached is a consonant then go back 3 letters (-3)".

Starting from *A* and applying the above cited rules, what is the letter that corresponds to the 96th position?
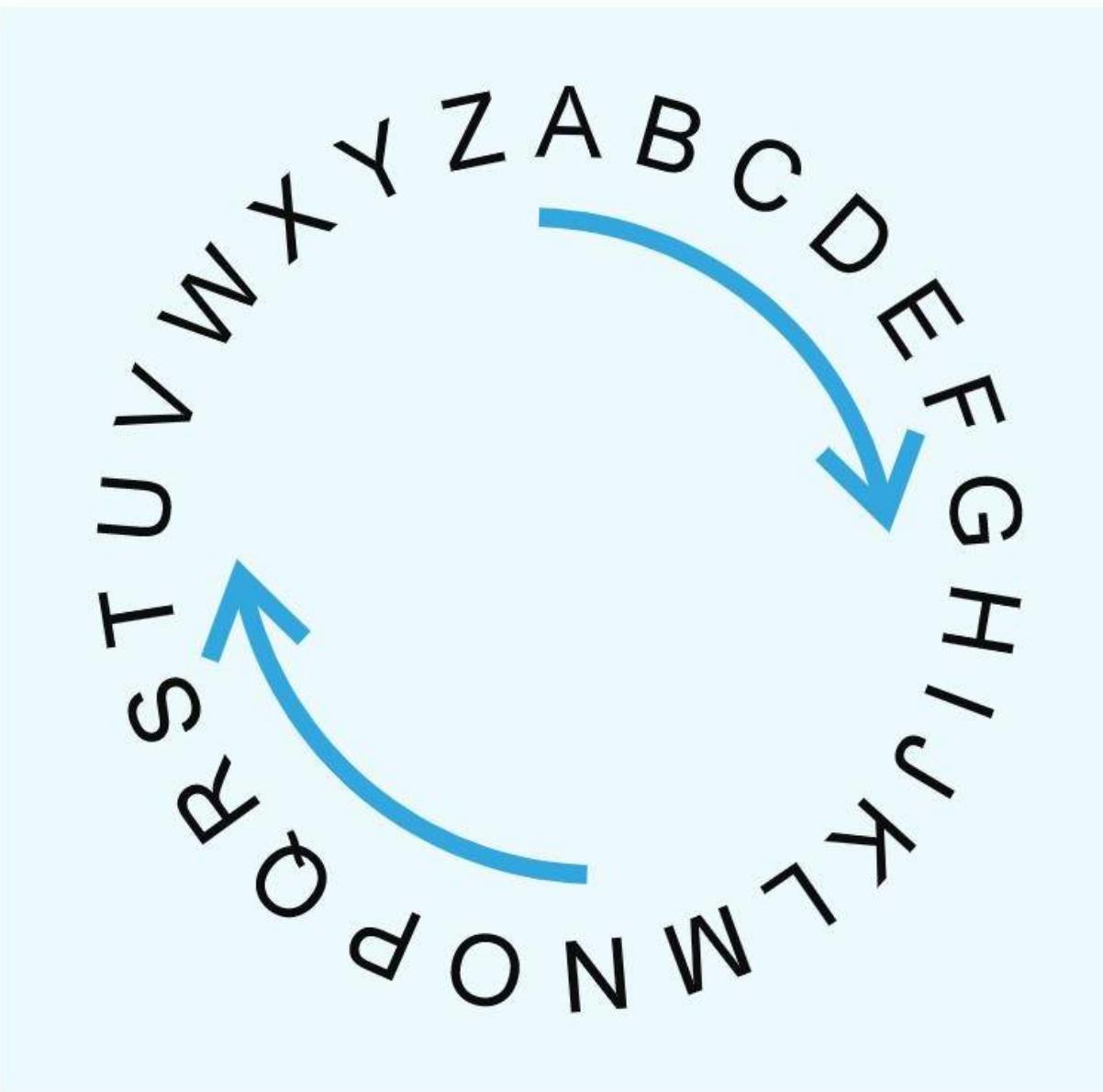
Comparison of signed numbers

Objectives

At the end of this chapter, I will be able to compare signed numbers.

Activities
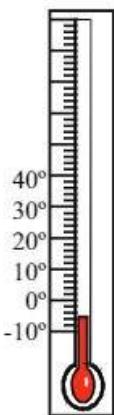
A) While spending a week in Faraya, a group of students have recorded the temperature in °C and obtained the following data:

Day	Temperature(°C)
Sunday	+6.5°
Monday	+4°
Tuesday	0°
Wednesday	-3.2°
Thursday	-4°
Friday	+2°
Saturday	+5.4

It is very hot today!

It is very cold today!
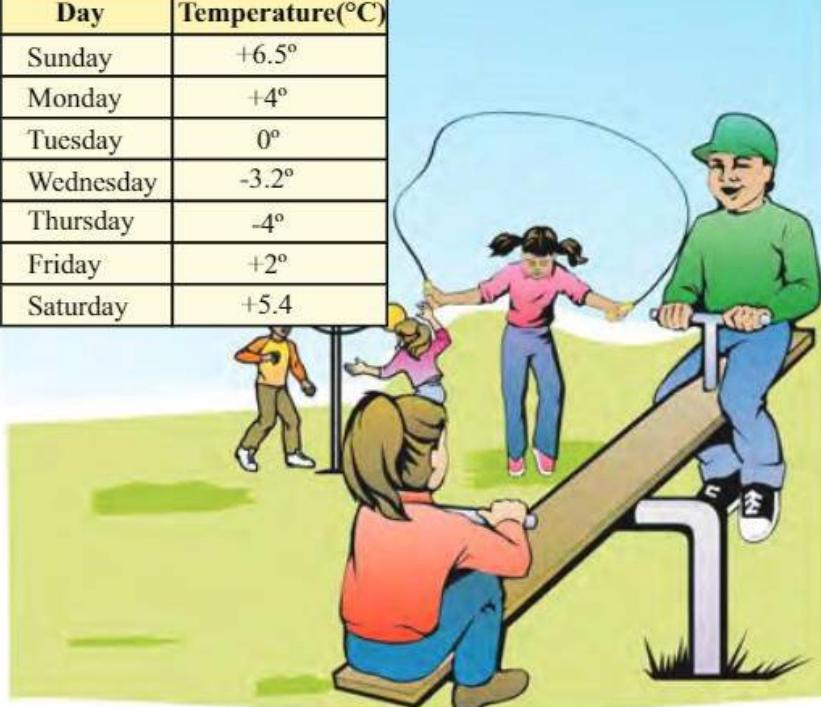
- Indicate the days of the week in which the temperature was above zero, then those in which the temperature was below zero.

- On which day was the temperature higher:

a) Saturday or Friday?

b) Wednesday or Tuesday?

c) Sunday or Tuesday?

d) Tuesday or Wednesday?

e) Monday or Wednesday?

f) Wednesday or Sunday?

● Copy and complete with the sign > or < :

+5.4..............+2	+6.5.............0	+4..............-3.2
-3.2..............-4	-3.2.............0	-3.2.........+6.5

B) Reproduce
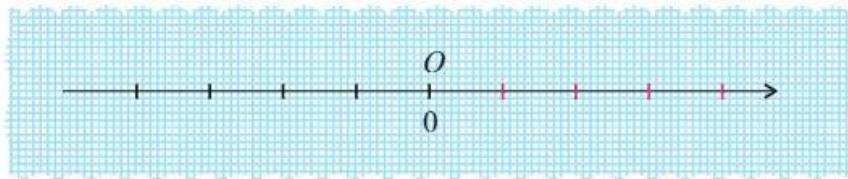
● Locate on this axis the points A; B; C; D; E; F and G of abscissas:

+6.5;   +4;   0;   -3.2;   -4;   +2;   +5.4.

● Copy and complete the following steps in the model:

+6.5 is a positive number; we write $+6.5 > 0$.

6.5 is at the right of zero.

+4 is a .... number   ; we write ....

4 is at the .... of zero.

-3.2 is a .... number   ; we write ....

3.2 is at the .... of zero.

-4 is a .... number   ; we write ....

4 is at the .... of zero.

+2 is a .... number   ; we write ....

2 is at the .... of zero.

+5.4 is at the .... number ; we write ....

5.4 is at the .... of zero.

● Put these numbers in an increasing order:

+6.5; +4; 0; -3.2; -4; +2; +5.4.

● Which is greater: -4 or +4?

● Locate on the axis the points: $E(x)$; $F(y)$; $G(z)$: such that:

$x > +3$; $y < +2$ and $-1 < z < +2$.

● Mark the part of the axis where we can place:

  - E (in blue)

  - F (in red)

  - G (in green).

  a being a signed number: $-1.5 \leq a \leq -0.3$.
  Stripe the segment on which H, of abscissa a, can be

$+2 < +3$

$-3 < -2$

$+1 > -2$

$+2 > 0$

$-1 < 0$
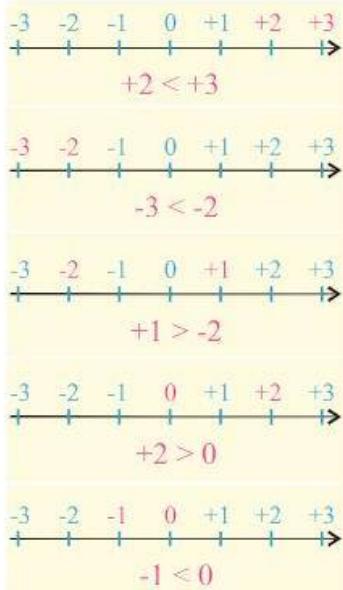
- **In order to compare signed numbers**, we place them on a numerical axis: The greatest should be the farthest to the right.

- If two signed numbers are :

  - positive, the greater is the farther from zero;

  - negative, the greater is the nearer to zero;

  - one positive and the other negative, the positive is greater.

- Every positive number is greater than zero.

- Every negative number is less than zero.

Focus

- **Comparison of a signed number a to 0.**

if a is positive, then $a \geq 0$ ← $a \;?\; 0$ → if a is negative, then $a \leq 0$

- **Comparison of two signed numbers a and b.**

$a \;?\; b$

if a and b are two positive numbers, then the smaller is the one which is nearer to zero

if a and b are of opposite signs, then the negative one is the smaller.

if a and b are two negative numbers, then the smaller is the one which is farther from zero.

$+1 < +2$

$-1 < +1$

$-2 < -1$
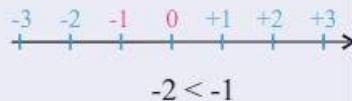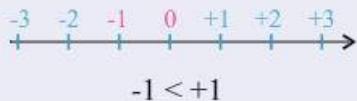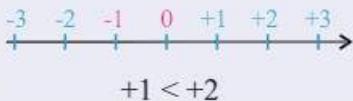
1- Choose from the following numbers:

-14;   +8; 0; +7.2;   -6.4;   -5.2;

+6;   -3;   -4.2;   +5.31;   -6.04;

+15;   -17.0; +53;   -9.4;

• the integers.

• the signed negative numbers.

• the signed positive numbers.

2- Fill in with < or > :

+5.2 0		+5.4 .... -5.40	
-3.4 0		-7 .... +7	
+6 .... -2		-14.25 .... -2	
-2 .... +2		-73.4 0	
-4.2 .... 6.5		-7 .... -4.	

3- Choose from each given list the greatest and the smallest signed number:

a) -13; +12; 0; +5.3; -12.9; +5.4; +12.9.

b) +2.7; +2.9; -3; +3; -4.5; +5.2; +4.50.

c) +7.3; +7.33; -7;   -7.03; -7.033; +7.03; +7.033.

4- Locate in each of the cases below the numbers on a numerical axis, then arrange them in an increasing order:

a) +2.5   ;   -4.21   ;   +4.2 ; +1.3 ;

   -2.5   ;   -1.2   ;   +4.21; -5.6.

b) +9.2; +9.22; -7.3; -7.5;

   0; +5.1; +5.15.

5- In each of the cases below put the numbers in decreasing order:

a) -3.14 ;   -27   ; -31.14 ;
   -3.1 ;   -31.04   ; -31.42 .

b) -19   ;   -3.13   ; -19.5 ;
   -19.51 ;   -3.01   ; -3.10 .

6- Give all the possible values of the signed number a in each of the following cases:

   1°) $+2 < a < +5$

   2°) $+2 \leq a \leq +5$

   3°) $+2 < a \leq +5$

   4°) $+2 \leq a < +5$.

7- Let b be an integer such that

   $0 \leq b < +7$.

Locate on the numerical axis all points that can represent b.

Repeat the same for:

   $-3 < c \leq 5$

   $-4 \leq d \leq 0$

   $-2 < e < +2$.

8- Let t; u and v be whole signed numbers:
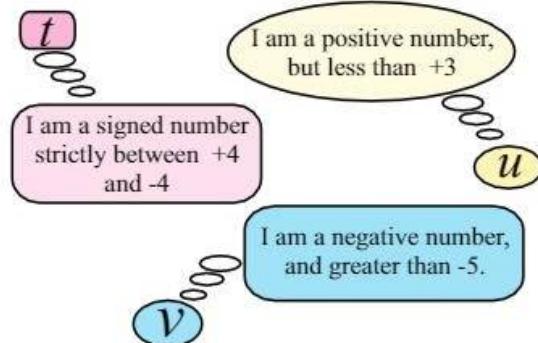
Locate on the numerical axis the points that can represent t; u and v.

1- Copy and fill in with the signs < or >:

        -7.2   ....   -7.5 ;   -16.03   ....   +13.06.

        +2.25   ....   +2.35 ;   +5.1   ....   -19.49.

2- Arrange in an increasing order the following numbers:

  -6.3 ; +6.03   ; +7.17   ; +7.1 ; -6.03 ; +6.33   ; -6.33.

3- Write all the possible values of x if:
   - x is a whole number;

        and
   - x verifies the following inequalities: $x \leq 0$ and $x \geq -10$.

Problems

1- During a given week, Sami recorded the temperature of each morning and obtained the following data:

Day	Temperature
Monday	+5°C
Tuesday	+2°C
Wednesday	-2°C
Thursday	+4°C
Friday	-6°C
Saturday	0°C
Sunday	+3°C

Arrange these temperatures in decreasing order, then indicate the day in which the temperature was lowest.

2- Find a negative number "a" knowing that:
   - a is made up of 3 digits:
     1; 4 and 8;
   - none of the digits is repeated;
   - $-8.14 < a < -1.48$.

3- In a math test, the difference between the grades and 10 were as follows:

      +5; -2; -0.5; +6; +0.5; -3; +4.

Put these numbers in an increasing order.
Write the grades of the seven students over 20.

4- A mother took the temperature of her feverish child every morning and evening for three days:

	Thursday	Friday	Saturday
Morning	38°C	37.9°C	37.5°C
Evening	39.2°C	39.9°C	37.9°C

When was the child most ill?

Understand then complete the series of numbers:

a) -5.25;   -4.25;   -3.25;   ..... ;   .....;   ;   .....;   ..... .

b) +17.32;   +13.32;   +9.32;   ..... ;   .....;   ;   .....;   ..... .

c) +6.303;   +6.404;   +6.505;   ..... ;   .....;   ;   ......;   ..... .

Multiplying and dividing fractions

Objectives

At the end of this chapter, I will be able to:
- ■ Multiply two fractions.
- ■ Divide two fractions.

Activities

Activity 1
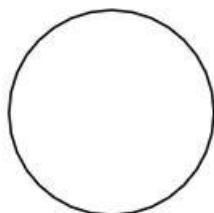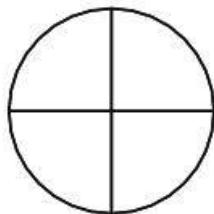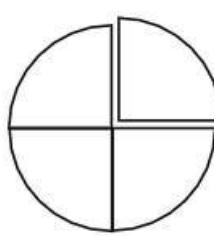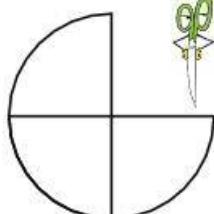
A) a) Draw a circle and cut the disc surrounded by this circle.

a)

b) Divide the disc into four equal parts by folding it.

c) Cut into four parts?

d) Color $\frac{2}{3}$ of the remaining portion red and express it by a fraction of denominator is 12 then, simplify this fraction.

b)

e) Complete the following:

$$\frac{2}{3} \times \frac{3}{4} = \frac{\ldots}{12} = \frac{\ldots}{\ldots}$$

c)

$$\frac{3}{4}$$
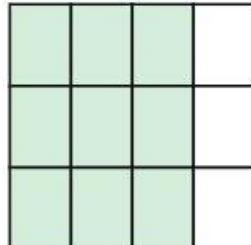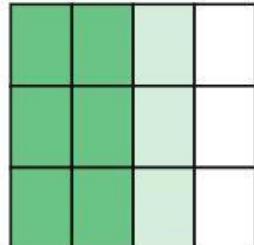
$$\frac{2}{3} \text{ out of } \frac{3}{4}.$$

f) Repeat this activity with the rectangle $ABCD$:

d)
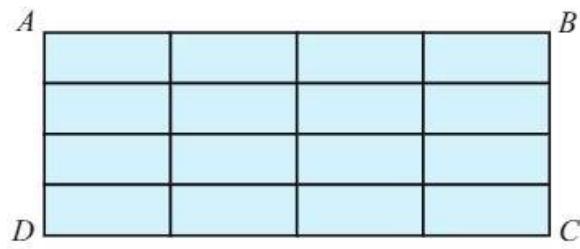
B- Hussam and Rola went biking from city A to city B which is 15 km away from city A.

But Hussam didn't ride except $\frac{1}{5}$ of $\frac{2}{3}$ of the distance because his bike was damaged. Rola couldn't ride except $\frac{2}{3}$ of $\frac{1}{5}$ of the distance because she was tired.

a) Draw two segments of 15 cm in length.

Houssam: ├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤

Rola: ├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤

Use them to denote the two fractions of the distance of denominator 15 cut by Hussam and Rola.

b) Complete the following:

$$\frac{1}{5} \times \frac{2}{3} = \frac{....}{15} \qquad\qquad \frac{2}{3} \times \frac{1}{5} = \frac{....}{15}$$

Compare the two fractions to know who rode more: Hussam or Rola.

Activity 2

A mother made a pie and divided it into 8 equal parts. She found out that only 6 parts were good for eating, so she distributed them equally among her three children.

a) Show the share of each child by a fraction of denominator 8.

b) Complete the following: $\dfrac{6}{8} \div 3 = \dfrac{....}{....}$
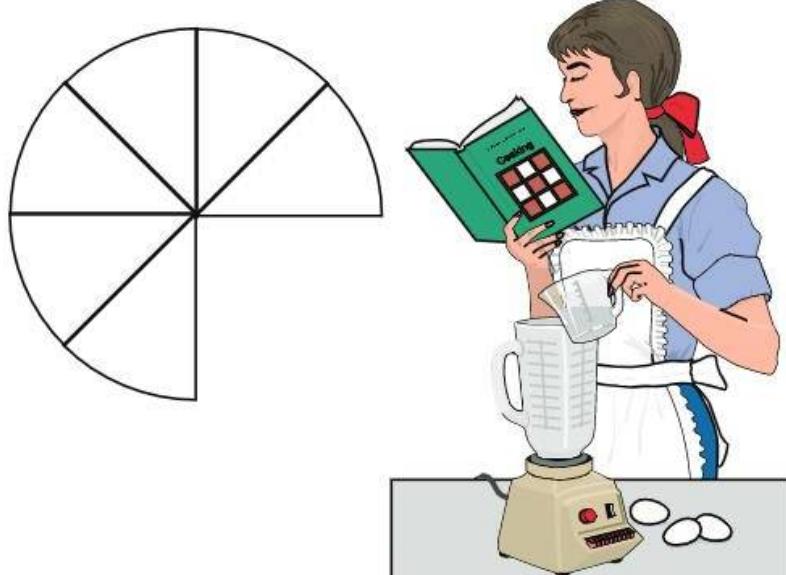
Compare between: $\dfrac{6}{8} \div 3$ and $\dfrac{6}{8} \times \dfrac{1}{3}$.

1. To multiply two fractions $\frac{2}{7}$ and $\frac{8}{9}$ we do the following:

- we multilply the numerators of the two given fractions

  $2 \times 8 = \boxed{16}$ ⟵ —— the numerator of the **product** ;

- we multiply the denominators of the two given fractions

  $7 \times 9 = \boxed{63}$ ⟵ —— denominator of the **product** ;

- **Then the product fraction is** $\boxed{\frac{16}{63}}$.

The operation is as follow $\boxed{\frac{2}{7} \times \frac{8}{9} = \frac{2 \times 8}{7 \times 9} = \frac{16}{63}}$.

To multiply 2 mixed fractions

$$2\frac{1}{3} \times 3\frac{1}{4} ,$$

- we change them into fraction:

$$2\frac{1}{3} = \frac{(2 \times 3) + 1}{3} = \frac{7}{3}$$

$$3\frac{1}{4} = \frac{(3 \times 4) + 1}{4} = \frac{13}{4}$$

- then multiply.

$$\frac{7}{3} \times \frac{13}{14} = \frac{91}{12}$$

Division is an opposite operation to multiplication.

Reciprocal of the fraction $\frac{7}{5}$ is a fraction $\frac{5}{7}$.

Two fractions whose product is one are reciprocal.

To divide two mixed fractions
- we change them into fractions ,
- then we divide.

2. To divide two fractions whose denominators are different from Zero, like $\frac{3}{11}$ and $\frac{2}{5}$, we multiply the first fraction with the reciprocal of the second fraction:

$$\frac{3}{11} \div \frac{2}{5} = \frac{3}{11} \times \frac{5}{2} = \frac{5 \times 3}{11 \times 2} = \frac{15}{22} .$$

The product of two fractions is a fraction whose numerator is the product of the numerators of the two fractions and whose denominator is the product of their denominators.

The denominators of two fractions should be different from Zero.

Example: $\dfrac{2}{9} \times \dfrac{4}{7} = \dfrac{2 \times 4}{9 \times 7} = \dfrac{8}{63}$.

The quotient of two fractions is equal to the product of the first fraction by the reciprocal of the second.

Example: $\dfrac{5}{7} \div \dfrac{3}{2} = \dfrac{5}{7} \times \dfrac{2}{3} = \dfrac{5 \times 2}{7 \times 3} = \dfrac{10}{21}$.

Exercises

1- Find the product of each of the following:

$\dfrac{2}{7} \times \dfrac{4}{9} = \dots$ $\dfrac{3}{8} \times 1 = \dots$

$\dfrac{4}{5} \times \dfrac{5}{4} = \dots$ $4 - \dfrac{4}{9} \times 0 = \dots$

$2\dfrac{3}{7} \times \dfrac{1}{2} = \dots$ $\dfrac{2}{11} \times \dfrac{199}{199} = \dots$

$\dfrac{7}{9} \times \dfrac{7}{9} = \dots$ $4\dfrac{3}{7} \times 3\dfrac{1}{5} = \dots$

$\dfrac{2}{3} \times \dfrac{1}{10} = \dots$ $6\dfrac{1}{2} \times \dfrac{2}{13} = \dots$

2- Find the quotients:

$\dfrac{5}{8} \div 4 = \dots$ $\dfrac{7}{9} \div \dfrac{1}{2} = \dots$

$\dfrac{4}{9} \div 2\dfrac{1}{4} = \dots$ $\dfrac{2}{9} \div \dfrac{2}{9} = \dots$

$12\dfrac{1}{2} \div 1\dfrac{3}{4} = \dots$ $\dfrac{6}{7} \div 13 = \dots$

$\dfrac{2}{3} \div 10 = \dots$ $\dfrac{5}{6} \div \dfrac{1}{10} = \dots$

$2\dfrac{8}{9} \div 4\dfrac{4}{9} = \dots$ $25 \div \dfrac{25}{2} = \dots$

3- Copy and complete:

$\dfrac{2}{3} \times \dfrac{\dots}{\dots} = \dfrac{6}{10}$ $\dfrac{\dots}{\dots} \times \dfrac{5}{3} = \dfrac{15}{21}$

$\dfrac{3}{4} \times \dfrac{\dots}{7} = \dfrac{6}{\dots}$ $\dfrac{3}{\dots} \div \dfrac{2}{\dots} = \dfrac{15}{8}$

$\dfrac{\dots}{13} \div \dfrac{\dots}{3} = \dfrac{9}{26}$ $\dfrac{3}{8} \div \dfrac{\dots}{7} = \dfrac{\dots}{32}$

$\dfrac{\dots}{4} \div \dfrac{5}{3} = \dfrac{9}{\dots}$

4- Choose the right answer:

$\dfrac{3}{7} \times \dfrac{1}{10} = ?$ $\dfrac{30}{70}$ $\dfrac{3}{70}$ $\dfrac{3}{7}$

$\dfrac{4}{13} \times \dfrac{13}{4} = ?$ 1 $\dfrac{4}{13}$ $\dfrac{13}{4}$

$1 \div \dfrac{5}{4} = ?$ $\dfrac{5}{4}$ 1 $\dfrac{4}{5}$

$\dfrac{2}{3} \div \dfrac{3}{2} = ?$	1	$\dfrac{2}{3}$	$\dfrac{6}{3}$
$2\dfrac{1}{2} \div 1\dfrac{1}{4} = ?$	$1\dfrac{1}{4}$	$1\dfrac{1}{2}$	2
$\dfrac{7}{9} \div \dfrac{1}{10} = ?$	$\dfrac{7}{90}$	$\dfrac{70}{90}$	$\dfrac{70}{9}$

a) $\dfrac{2}{7} \times \dfrac{3}{8} =$

b) $\dfrac{3}{8} \times \dfrac{2}{7} =$

a) $\dfrac{3}{5} \times \left(\dfrac{1}{2} - \dfrac{9}{10}\right) =$

b) $\left(\dfrac{3}{5} \times \dfrac{1}{2}\right) \times \dfrac{9}{10} =$

5- Complete and simplify, then compare between answers A and B in each of the following cases:

a) $\dfrac{2}{3} \times \left(\dfrac{2}{3} - \dfrac{1}{2}\right) =$

b) $\dfrac{2}{3} \times \dfrac{2}{3} - \dfrac{2}{3} \times \dfrac{1}{2} =$

$\dfrac{2}{3}$

$\dfrac{2}{3} - \dfrac{1}{2}$

$\dfrac{2}{3} \times \left(\dfrac{2}{3} - \dfrac{1}{2}\right)$
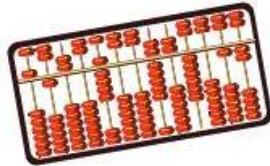
a) $3\dfrac{2}{5} \times \left(\dfrac{3}{22} + \dfrac{0}{11}\right) =$

b) $3\dfrac{2}{5} \times \dfrac{3}{22} + 3\dfrac{2}{5} \times \dfrac{5}{11} =$

a) $\dfrac{2}{3} \div \left(\dfrac{5}{3} \div \dfrac{1}{2}\right) =$

b) $\left(\dfrac{2}{3} \div \dfrac{3}{5}\right) \div \dfrac{1}{2} =$

Self-evaluation

Correct the following operations:

$\dfrac{10}{11} \times \dfrac{5}{11} = \dfrac{50}{11}$ $\dfrac{3}{13} \times \left(\dfrac{3}{4} - \dfrac{2}{3}\right) = \dfrac{3}{13} \times \dfrac{3}{4} = \dfrac{2}{3}$ $2\dfrac{1}{3} \times 3\dfrac{1}{2} = 6\dfrac{1}{6}$ $\dfrac{17}{4} \times \dfrac{17}{3} = \dfrac{17}{12}$.

Problems

1- During a car race Raji drove $\dfrac{4}{7}$ of the required distance and Rajaa drove $\dfrac{2}{3}$ of the distance Raji drove.

What is the fraction of the distance Rajaa drove?

2- An employee spends $\dfrac{8}{9}$ of his monthly salary. If he spent $\dfrac{3}{7}$ of his spent money on food, write a fraction to show how much he spent on food.

3- Consider a rectangle of area $18\dfrac{2}{3}$ m^2 and width $2\dfrac{1}{3}$ m. Calculate its length and its width.

4- Rima ate $\frac{1}{4}$ of $\frac{2}{3}$ of a cake and her brother Adel $\frac{2}{3}$ of $\frac{1}{4}$ of this cake.

a) What is the fraction of the cake that Rima and Adel ate?

b) Who ate more cake?

5- Mirna wants to distribute $2\frac{1}{3}$ liters of milk in glasses of capacity $\frac{7}{9}$ liter each. How many glasses can she fill?

6- A worker wanted to tile two rooms with a total area of 54 m^2. On the first day, he tiled $\frac{7}{9}$ of the floor in the first room. On the second day, he tiled $\frac{7}{9}$ of the second room. What is the area tiled in 2 days?

7- Nada paid 26000 LL for the price of $3\frac{1}{4}m$ of cloth.

What is the price of buying 1 m of this cloth?

8- A ship carries $6\frac{2}{5}$ tons of woods, its cargo was distributed among trucks that have a capacity of $\frac{4}{5}$ ton each.

What is the number of trucks needed to carry this cargo?

9- What is the nearest number to 30 whose $\frac{1}{9}$ of it is a natural number?

Calculate $\frac{1}{9}$ of this number.

10- A merchant bought $\frac{3}{4}$ of a cargo of apple and sold half of it.

Write a fraction to show the quantity of apples he sold.

**11-** What is the area of a disc whose radius is $2\frac{3}{7}$ cm .($\pi = 3.14$)

Give the answer in the form of a mixed fraction.

12- The side length of a square $ABCD$ is $3\frac{2}{5}$ cm. Let M be a point on $[CD]$ such that $MC = 2cm$.

a) Calculate the area of $ABCD$.

b) Calculate the area of triangles BCM, AMB.

c) Consider the fraction whose numerator is equal to the area of AMD and whose denominator is equal to the area $ABCD$. Simplify this fraction.
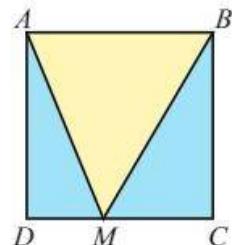
13- Half the number of boys and half the number of girls in a school forms the internal section. If $\frac{2}{3}$ of the students in this school is boys:

a) - What fraction represents the number of girls in the internal section?

- What fraction represents the number of boys in the internal section?

b) What is the number of girls in the internal section if the number of the students in the school is 324 students?

Just for fun

Show in the two figures below how:

$$\frac{2}{5} \times \frac{3}{4} = \frac{3}{4} \times \frac{2}{5}$$
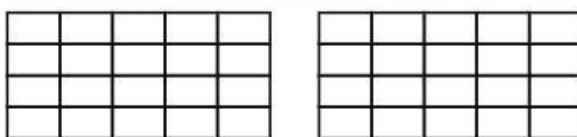
Quotient and ratio

Objectives

At the end of this chapter, I will be able to:

- [ ] recognize and represent the quotient of a division.
- [ ] recognize and use the ratio of the two extensions.

Activities

Activity 1

$$\frac{1}{3} = 0.333333333...$$

The quotient of 1 by 3 is a number made of a hole part and an unlimited decimal part of representative number 3.

$$0.3 < 0.333...... < 0.4$$

Is the enclosing of 0.3333...... between 2 decimal numbers of 1 decimal part each.

$$\frac{46}{33} = 1.39393939......$$

a) In each of the following cases, write the quotient of a by b in the form of a fraction:

$a = 932$	$a = 5$	$a = 4$	$a = 42$	$a = 6$
$b = 23$	$b = 4$	$b = 3$	$b = 33$	$b = 5$

b) Copy the tables given below:
Arrange each of the quotients you have obtained in the appropriate place in the tables:

Natural Number	Decimal Fraction	Decimal Number	No decimal Fraction	Decimal Writing
....	....	....	....	....

c) Enclose the quotient $\frac{4}{3}$ between:

1) two consecutive natural integers.
2) two decimal numbers with one decimal part each.
3) two decimal numbers with two decimal parts each.
4) two decimal numbers with three decimal parts each.

d) Can you determine the 70th decimal part of the quotient of 4 by 3?

e) How can you represent a quotient and a writing that is useful in all cases?

Activity 2
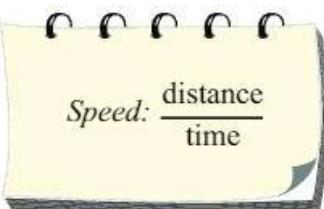
A) To make a chocolate cake we used the following ingredients:

Cocoa 75 g ; Sugar 260 g ; Flour 0.3 kg ; Butter 0.2kg

$\frac{5}{8}$ is the ratio of 5 to 8.50 ratio is a comparison of 2 numbers or two quantities of the same unit; otherwise, I should find the unit.

Eg: to find the ratio of 500 g to 2 *kg*, I should convert the 2 *kg* to 2000 g.

The Lebanese territory has a surface area of 10452 km². If the number of inhabitants is 3895000, then the average population is $\frac{389500}{10452}$ inhabitants/km^2.

Give the following ratios:

a) That of the mass of cocoa to the mass of sugar.
That of the mass of sugar to the mass of butter
Compare these two ratios.
What do you notice?
Do these ratios have units?

Speed: $\frac{distance}{time}$

b) A bus coverd a distance of 300 *km* in 6 *hours*, moving at a constant speed to visit Lebanese ruins.
This same school-bus had previously covered a distance of 400 *kms* in 8 hours also moving at a constant speed.
Find the average speed of the bus per hour in each of the two trips.
Compare the two answers showing your reasoning.

Text

The quotient:

To find the quotient of the division of two numbers a and b, we can apply the common method used in division. This quotient can be:

• a natural number
    or
• a decimal one: if you can write $\frac{a}{b}$ as a decimal fraction.

See the examples given next.

a	b	quotient
225	25	$\frac{225}{25} = 9$
22.5	2.5	$\frac{22.5}{2.5} = \frac{225}{25} = 9$
2.25	2.5	$\frac{2.25}{2.5} = \frac{22.5}{25} = \frac{225}{250} = \frac{9}{10} = 0.9$
2.25	25	$\frac{2.25}{2.5} = \frac{225}{2500} = \frac{9}{100} = 0.09$
2.25	250	$\frac{225}{250} = \frac{225}{25000} = \frac{9}{1000} = 0.009$

- a decimal number in which the decimal part is infinitely repetitive so $\frac{a}{b}$ is a non-decimal fraction, as is the case for $\frac{15}{11}$ since

$$\frac{15}{11} = 1.3636......$$ 3 and 6 will be repeated continuously.

- Given an approximate value for example the quotient of 2 by 3 is 0.666..... so we can:

  - Enclose 0.666...... between two natural numbers or two decimal numbers within a range of 1 or 0.1 according to the required precision

Example: $0 < 0.666.... < 1$
                $0.6 < 0.6666.... < 0.7$
                $0.066 < 0.6666..... < 0.67$
                $0.666 < 0.666..... < 0.667$

  - Consider the first number in the enclosure as being an approximate value in deficit and the other an approximate value in excess.

- Written as $\frac{a}{b}$ to represent a quotient in all cases on condition that b ≠ 0.

Ratio

- The fraction $\frac{4}{5}$ is the ratio of 4 to 5. In general, the ratio of a to b is the fraction $\frac{a}{b}$, with b different from zero.

- If the two numbers to be compared are of the same type (like two lengths, or two areas, or two ages, or two weights...).

Example: If the length of a bar is 3 cm, and the length of a piece of wood is 7cm, then the ratio of the length of the bar to that of the piece of wood is $\frac{3}{7}$.

- If the two numbers to be compared are of the same type but of different subdivisions of units like the cm and m for length, the g and kg for mass,.... you should convert them to the same unit then compare.

Example: If Wadad is 10 years old and Samir is 145 months old, you convert the age of Wadad to months.

⇒Wadad = 10×12 = 120 months.
⇒ratio of Wadad's age to that of Samir's is $\frac{120}{145} = \frac{24}{29}$.

- The ratio is one of the methods used in comparison:

If $\frac{a}{b}$ is greater than a unit (one), this means that a is greater than b and vice-versa.

In this case, the ratio is unitless.

- If a and b are of different units (example if (a) is a measure of time and (b) a measure of length), then the unit of the ratio is the unit of (a) divided by the unit of (b).

- If the ratio is constant, then we call it average.

I simply say that the quotient of 2 by 3 is $\frac{2}{3}$

Example: If the price of 4 meters of fabric is 20000LL then:

$$\frac{\text{Price of fabric}}{\text{N}^\circ \text{ of meters of fabric}} = \frac{20\ 000\ \text{LL}}{4\ m} = 5\ 000\ \text{LL}/m$$

Focus

The quotient of a by b

$\downarrow$

$\frac{a}{b}$; $b \neq 0$

decimal fraction

non-decimal fraction

The quotient is a natural number
or
The quotient is a decimal number

The quotient is a decimal number in which the decimal part is repetitive like

$$\frac{13}{11} = 1.181818...$$

(1 and 8 are repetitive)

the average value is
→ 1.1 (default)
↘ 1.2 (excess)

Ratio

$\downarrow$

ratio of a to $b = \frac{a}{b}$; $b \neq 0$

a and b are of the same unit

a and b are of different units

ratio is unitless

ratio has a unit

$$\frac{a}{b} < 1 \Leftrightarrow a < b$$

$$\text{unit} = \frac{\text{unit of } a}{\text{unit of } b}$$

if ratio is a constant then we call it average.

Exercises

1- Find the quotient of a by b in each of the following cases:

$a = 1353$	$b = 11$
$a = 310.5$	$b = 9$
$a = 69.44$	$b = 12.4$.

2- Give the repetitive numbers (frequency) in the quotient of a by b in each of the following:

$a = 16$	$b = 3$
$a = 67$	$b = 66$
$a = 368$	$b = 999$.

Check your answers using a calculator.

3- Find the approximate value for the quotient of a by b based on the following conditions:

$\dfrac{150}{26}$ to the nearest 0.1 in excess;

$\dfrac{123}{7}$ to the nearest 0.01 by default;

$\dfrac{25}{6}$ to the nearest 0.001 in excess.

4- Find the approximate value of the quotient of the division of 81458.23 by 7175.256 to the nearest 0.001 in excess. (you may use the calculator).

5- Give the quotient $\dfrac{a}{b}$ in fractional form and simplify it if possible:

$$\dfrac{145}{120} \quad ; \quad \dfrac{68}{17} \quad ; \quad \dfrac{0.1}{0.01} \ .$$

6- Given a rectangle that is $60\,mm$ in length and $3\,cm$ in width .
- Find the ratio of length to width;
- Find the ratio of width to length;
- Find the ratio of perimeter to surface.

7- Given a square with a side equal to $4\,cm$. Find the ratio of the side of this square to its perimeter;
Find the ratio of the side of this square to its area.

8- Fadi covers by bike a distance of $40\,km$ in two hours, at a constant speed. Find the average speed of Fadi per hour.

9- Look at the given example.
The speed of a turtle is equal to $108m$ per hour

$$\dfrac{108 \times 100\ cm}{3\,600 \times 1\ s} = \dfrac{10\,800}{3\,600} = 3\ cm/s$$

If the speed of the eagle is $162\ km/hour$, convert this speed to m/s.

10- Circle the correct answer:

If $\dfrac{a}{b} = 0.9$, then:

$a > b$	$a = b$	$a < b$

Self-evaluation

- Find using your calculator the approximate value of the quotient of the division of the two numbers: 1 234 567 and 532.123 to the nearest 0.001 in excess.

- If the ratio of the area covered by water on Earth to the area of the land is equal to $\dfrac{71}{29}$ whose area is greater: that of land or that of water? Justify.

1- The ratio of the number of girls in the 6th grade to that of the boys in the same class is $\frac{5}{6}$. Which is greater, the number of girls or boys? Why?

Find the approximate value for the quotient of the division of two numbers 5 and 6 to the nearest 0.01 in default, then compare this value with a unity.

2- The diameter of Pluto is approximately $3 \times 10^3 km$ and that of Saturn is $120 \times 10^3 km$, approximately.

a- Find the ratio of the length of Pluto's diameter to that of Saturn.

b- Write this ratio as a decimal number, then as a decimal fraction.

3- If every 9 *kg* of sea water contains 288 *g* of salt, find the ratio of the mass of salt to water and transform this ratio into a decimal number.

4- If Sami is 156 months old and Samar is 21 years old, find the ratio of Sami's age to that of Samar's.

Find the average value of the quotient of the age of Samar to Sami to the nearest 0.1 in excess.

5- In 2 010, it is expected that the Lebanese's needs for water becomes 3 300 million m^3, of which 2 160 million m^3 will be used for irrigation.

a- Find the ratio of water needed for irrigation to the total amount of water Lebanon will need in 2 010.

b- Using your calculator, find the repetitive numbers of the quotient of the division of 3 300 million by 2 160 million.

c- Find the approximate value of this quotient of the nearest 0.001, in excess.

6- Sannine is 2 628 *m* high and Al-Kanaisa is 2 032 *m* high.

a- Find the ratio of height of Sannine to that of Al-Kanaisa. Write this ratio as an irreducible fraction.

b- Using your calculator, find the average value of the quotient of the division of 2628 by 2 032 to the nearest 0.001 by default.

7- The area of the Saudi territory is 2.2 million km^2, and the area of the Egyptian territory is 1 million km^2.

a- Find the ratio of the area of the Egyptian territory to the Saudi.

b- Find the quotient of the division of 2.2 million by one million and write it as a decimal fraction.

8- The human body is made up of 208 bones and 900 muscles.

a- Find the ratio of the number of bones to muscles.

b- Using your calculator, find the quotient of the division of 208 by 900 to the nearest 0.001 by default.

9- A train covers 1170 *km* in 9 hours.

a- Find its average speed per hour.

b- What distance will this train cover in 10 hours?

c- Find its average speed in meters/minute.

Just for fun

1. Find the perimeter and area of each of the squares given next.

2. Find the ratio of the perimeter to the length of the side of the square.

3. Find the ratio of the perimeter of each square to the area of the same square.
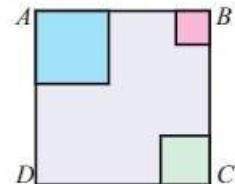
Percentage

Objectives

At the end of this chapter, I will be able to:
- calculate the percent age of a number ;
- recognize, calculate and compare percentages.

Activities

After spending a day shopping, a mother goes back home and calls her two children:

a) "Sara, take this dress. Its price was $300 before the sale, but I profited from a twenty- percent discount on its price.

Complete the table, by following the steps indicated and using your calculator, then you will know how much I paid".

Price	discount percent	discount on Price	Reduced price

b) "The T-shirt is yours Ziad; its price was $60 but I bought it for $45. The calculation shown in the following table will allow you to find the discount on this T-shirt".

Price	Reduced Price	Value of discount	Percentage of discount		
60	45	....	$\dfrac{\ldots\ldots - \ldots\ldots}{\ldots\ldots} = \ldots\ldots$		
			In the form of:		
			a decimal number	a fraction of denominator 100	percentage
			$\ldots\ldots =$	$\dfrac{\ldots\ldots}{100}$	$= \ldots\ldots \%$

Text

Percent of a number

To calculate one percent, two percent, three percent,.... of a number is to multiply it respectively by:

$$\frac{1}{100}, \qquad \frac{2}{100}, \qquad \frac{3}{100} \ldots\ldots$$

Example Five percent of $340 = \frac{5}{100} \times 340 = 17$.

We write: "17" is equal to "5 percent of 340".
better yet:

> 5% of 340 is equal 17.

> We say 2.5% to indicate the operation $\frac{2.5}{100}$ and avoid writing $\frac{25}{1000}$

• Percentages

The multiplication operators, $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, ...,

noted respectively as $\qquad$ 1%, 2%, 3%, ...,

are percentages.

• Comparison using percentages

Starting from an inequality $\qquad 620 < 734$

the operator "percent" $\qquad \frac{620}{100} < \frac{734}{100}$

8%

makes an inequality having the same significance (meaning), from where: $\quad 8 \times \frac{620}{100} < 8 \times \frac{734}{100}$

> The order is conserved when we apply a percentage.

> The multiplication with a positive number is compatible with the order.

• On the other hand, if you want to classify two numbers, m and n, of which 32 % are respectively 416 and 240, it is sufficient to compare 416 and 240. Since $416 > 240$, then $m > n$.

> Practically, to classify two numbers, it is sufficient to classify their "percent".

Determination of a "percentage" of a number:

To find the "percentage" of m represented by n:

you calculate the quotient $\frac{n}{m}$,

then you write this quotient as a fraction of denominator 100.

The numerator of the obtained fraction is the percentage we are looking for.

$m = 3500 \quad n = 420$

$\frac{420}{3\,500} = 0.12$

$0.12 = \frac{12}{100}$

$420 = 12\,\%$ de $3\,500$.

Examples:

- $\frac{35}{50} = 0.7$, therefore 35 is equal to 70 % of 50.

- $\frac{1\,573}{5\,720} = 0.275$, therefore 1 573 represents the 27.5 % of 5 720

Focus

3660

Then calculating $\boxed{3660} \times \boxed{35}\, \boxed{\%}$ $\boxed{183} \div \boxed{3660}\, \boxed{\%}$

We obtain: $\boxed{1281}$ $\boxed{5}$

The thirty-five percent of 3660

"183" represents the five percent of 3660

Exercises

1- Write as a percentage:

  a) 5 of 10   ;   1 of 4   ;

      2 of 25   ;   3 of 25 ;

      7 of 10   ;   1 of 50 ;

  b) 3 of 4   ;   9 of 20 ;

      30 of 50 ;   15 of 75;

2- Write as a percentage:

  a) $\frac{2}{40}$; $\frac{5}{16}$; $\frac{9}{32}$; $\frac{21}{20}$;

  b) $\frac{24}{60}$; $\frac{18}{15}$; $\frac{27}{75}$; $\frac{35}{175}$.

3- Write as a "percent":

  a) 0.2; 0.33; 0.15; 0.75; 0.80.

  b) 1.03; 2.25; 3.5; 7.35; 9.2.

4- What percentage of a disc do the following represent?

- the semi-disc?
- the quarter of a disc?
- the three quarters of a disc?
- the four fifths of a disc?
- the seven-twentieths of a disc.

5- The full central angle of a disc is divided into 6 angles of measures: 90°; 50°; 60°; 30°; 10° and 120°.

What percentage of the whole (full) angle do the above cited angles represent?
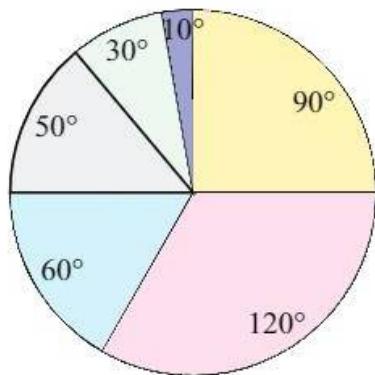
6- What is the 15 % of each of the following numbers:

130 ; 2 453 ; 3 202

5 500 ; 673.5 ; 87.42 -

9.990 ; 1.882 ; 88 880.

7- Calculate the 20 %, and the 30% of 18000.

Compare the sum of the two answers with the half of 18000.

Self-evaluation

1- State whether the following is true or false:

- 25 % of a number is its quarter (one fourth).

- 30 % of a number is its one third.

- 20 % of 250 = $\dfrac{250}{5}$.

- The eighth of a number is equal to 12.5 % of this number.

2- During sales, Samer and Rima bought two pairs of pants at half price. Samer paid 39000 LL for his and Rima paid 45000 LL.

Which of the two pairs of pants was more expensive before the sale?

Problems

1- A property has been bought for 35 000 000LL. We want, when reselling it, to make a profit of 7 %. What will the profit be?

2- A man bought 35 lambs for 135000 LL each. Since he paid the total amount in cash, he got a discount of 6%.
How much did he pay?

3- A merchant bought 175 meters of fabric for 2880 LL per meter. He sold the $\frac{2}{5}$ for a profit of 16 % and the remainder with a profit of 10 %.

How much did he gain?

4- A grocer buys 100 kg of green coffee for 13000 LL per kg. Knowing that, when green coffee is roasted, it loses 18 % of its mass, calculate the price of a kg of roasted coffee.

5- A farmer picked 3600 *kg* of olives to extract oil. Knowing that every 100 *kg* of olives makes 20 *kg* of oil, calculate the mass of oil this farmer will obtain. What in liters is the quantity obtained if one liter of oil weighs 0.90 *kg*?

6- A salesman buys a quantity of wheat, of which he sells the 2/5 for a price of 480 000 LL per ton. He thus gains 24 % on the buying price. He sells the rest of the wheat for 832700 LL thus losing 5000 LL per ton.
How many tons of wheat did he buy?

7- What is the price of an object labeled 10000 LL if you can benefit from a 30 % discount on its price?

8- A grocer buys 420 kilograms of coffee at 7350000 LL. How much should he sell the 200 g to get a profit of 11 % on the buying price?

9- A piece of fabric measures 47.86 m. When rinsed it loses 3.2 % of its length. What will its length be after rinsing?

10- An editor gives a 23 % discount on the price of packages. What is the discount done on a package of 30 000 LL?
What is the price of a package on which we got a discount of 17250 LL?

11- Milk gives 12 % of its mass as cream and 35 % of the mass of the cream as butter. How many kg of butter will 80 kg of milk give?

12- Since we cannot draw a ship with real measurements, we draw it by dividing all the dimensions by 1000. So we constructed this reduced model:
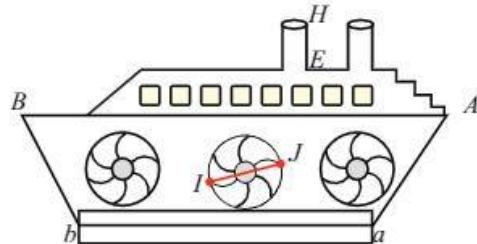
Take the necessary measures in your notebook and complete the following table:

Measure	CH	ab	AB	IJ
on the drawing in *cm*	......	......	......	......
in reality in cm	......	......	......	......

"We say that this drawing is made to the scale $\frac{1}{1\,000}$ which means that the measure of a length on this drawing is equal to $\frac{1}{1\,000}$ of the real measure of this same length.

$$\text{Scale} = \frac{\text{mesure on the drawing}}{\text{real measure}}$$

13- A rectangular field is represented by a rectangle of dimensions 3.2 *cm* and 5 *cm*, in a drawing with the indication:

$$\text{Scale } "\frac{1}{5000}".$$

Calculate the real dimensions of this field.

14- Points *A* and *B* on a geographical map represent two positions on Earth. The distance *AB* on the map represents the real distance (bird flight) between these two points. Calculate the real distances of :

* Bint Jbeil to Tripoli.

* Beirut to Baalbeck.

* Zahlé to Aley.

A football, a basketball and a tennis fields are represented below:
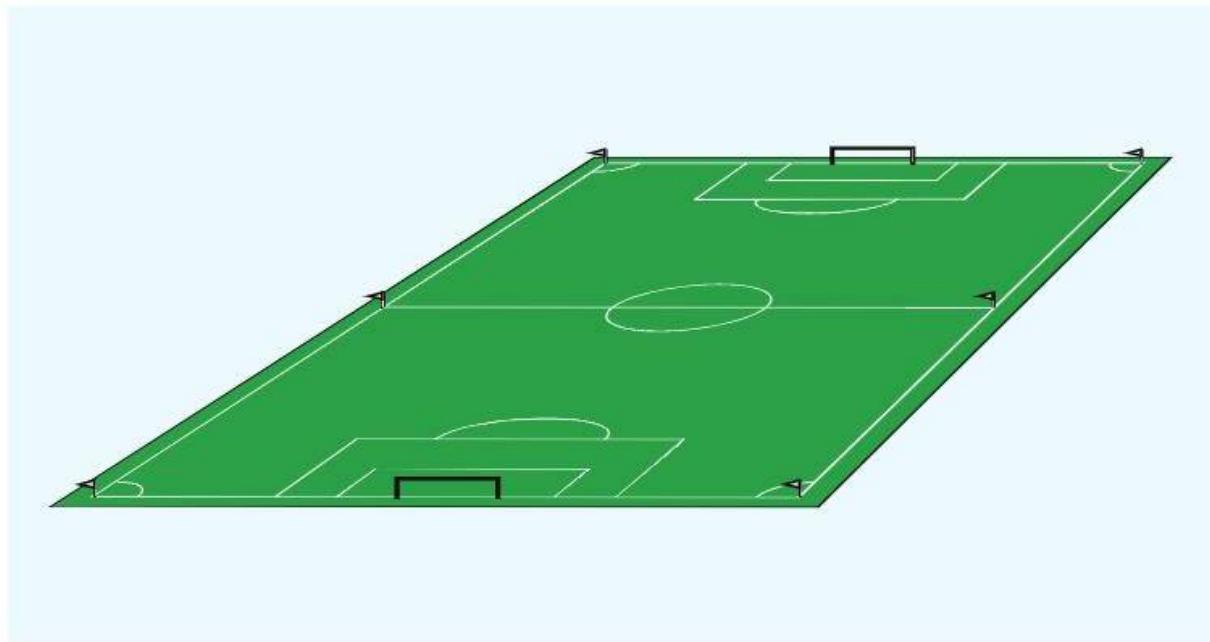
1) Knowing that a football field is a rectangle of dimensions 100*m* and 60*m* .
   Calculate the scale at which we drew the above field.

Proportionality

22

Objectives

At the end of this chapter, I will be able to:

- ☐ recognize and construct proportional chain (series);
- ☐ calculate the proportionality coefficient and the fourth proportional.

Activities
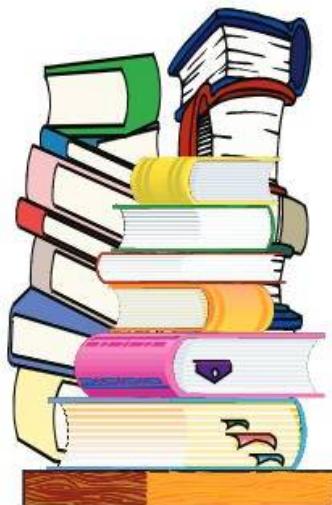
a) Make a parcel using two, three books and measure its height.

Using the above information, can you guess the height of the seven books without putting them in a parcel (on top of each other)?

b) Take several notebooks, all of the same thickness, and arrange them into parcels. For each parcel, write the number of notebooks or the height of the parcel in *cm*.

Reproduce and complete the table:

Number of notebooks	10	20	25	...	48	...
Height of the parcel	8	...	...	25.6	...	52.8

Determine the multiplication operator corresponding to each of the following steps:
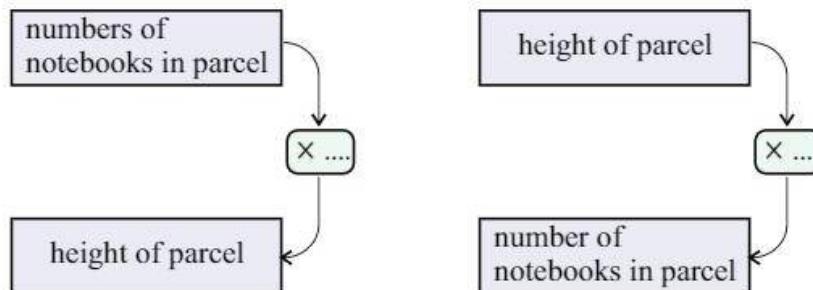
4	5	6
8	10	12

2	20
3	30
7	70
11.2	112

If there are 2 distinct coefficients, therefore there exists none!

0.3	0.8	1.2	7	10
1.2	3.2	4.8	28	40

6	7	9	11
7.8	9.1	11.7	14.3

6	7	9	11
7.8	9.1	11.7	14.3

The proportionality table

A numerical table with two lines is a proportionality table. If you multiply the numbers in one of the lines by a number "c" called the proportionality coefficient, you obtain the numbers of the other line.

Examples:

a- In the table given to the right, the products of the numbers given in the first line by "4" are the numbers given in the second line (row), therefore the given table is a proportionality table.

0.3	0.8	1.2	7	10
1.2	3.2	4.8	28	40

×4

b- As for the next table, no number can be put in the blank therefore this is not a proportionality table.

3	5	8
18	30	56

×...

Proportional Series   (Sequences)

If you reproduce the numbers of a proportionality table as they are and in the same order we obtain two sequences called "proportional series".

0.3	0.8	1.2	7	10
1.2	3.2	4.8	28	40

Also, when you have two proportional series you can arrange them in a proportionality table.

6	7	9	11
7.8	9.1	11.7	14.3

The proportionality coefficient

Given two proportional series, to calculate the proportionality coefficient is to divide one of the terms of the first series by the term having the same order in the second series.

$$9.1 \div 7 = 1.3$$
$$11.7 \div 9 = 1.3$$

Notice: If you have two proportional series, the coefficient will be the same no matter what you choose in terms of a different order.

The calculation of the coefficient can be a way to identify whether two series are proportional.

Examples:

9 - 16 - 2   and   12.6 - 22.4 - 3 $\dfrac{12.6}{9} = 1.4$ and $\dfrac{3}{2} = 1.5$

This indicates that the two series are not proportional.

$9 \div 2 = 4,5$

In fourth proportional

- If you are dealing with the small proportionality table "2 rows and 2 columns" and if one number only is missing, then this number is called the fourth proportional of the 3 other numbers.

  In order to determine the fourth proportional, it is sufficient to calculate the proportionality coefficient and let it act as an operator.

- Practically and in the case of a table

where a, b and c are known

the fourth proportional is: $x = \dfrac{a \times c}{b}$

Focus

We can go both ways

$5 \times 0.2 = 1$

Exercises

1- From the following table, choose the proportionality numbers:

5	6	7
12.5	15	17

3	5	7
3.6	6	8.4

10	15	40
4	6	16

4	3.2	7	8
12	9.6	21	24

18	30	9	105
3.6	6	1.8	21

14	18	21
15.4	19.8	23

2- Reproduce then complete the proportionality tables

× ....
4	7	9	....
0.8	1.4	....	2.2

× ....
18	.....	12.5	.....
72	40	.....	60

× ....
7	8	11
0.014	0.016	.....

3 %
38	50	77

4	5
0.12	....

....	3
21.7	9.3

3- Is there proportionality between the two lines in the following cases?

a- Age and length

age	10	11	12
length in cm	132	140	145

b- Quantity bought and price

number of roses	2	3	6
price in LL	500	750	1 500

c- Consumption and course

quantity of fuel consumed (in liters)	5	12	16
length of course in *(km)*	40	96	128

d- Number of steps and height reached

number of steps	4	7	10	12
height reached in *(cm)*	60	105	150	180

e- Number of pages and price

number of pages of a book	230	540	680
price in LL	3 000	7 000	10 000

Self-evaluation

a- Complete:

× ....   × ....
0.5	0.1	....	1.2	....
0.1	....	0.5	....	0.13

b- Find the fourth proportional:

2	....
7	35

....	1.2
20	24

4	11
12	....

6	0.08
....	0.4

Problems

1- When the big wheel effects 2 complete rounds, the small one effects five.
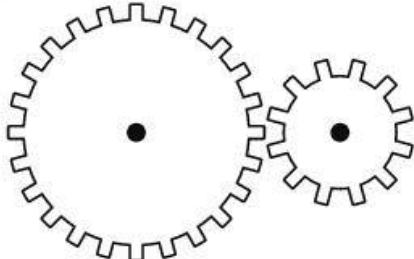
How many rounds will the small wheel complete if the big one does 32 rounds? How many rounds will the big wheel effect if the small one does 65?

2- A game consists of going backwards 2 steps wherever you advance (go forward) 7 steps.

- How many steps do you advance if you go backwards 50 steps? 120 steps? 150 steps?
- How many steps do you go backwards if you advance 35 steps? 49 steps? 105 steps?
- Where would you end (n° of final steps) if you made 180 steps in total?

3- To make a cake, we follow the directions given in the recipe below:

Sugar	Flour	Butter	Eggs
100g	75g	40g	3

Calculate in grams the necessary quantities of each ingredient if we want to make the same type of cake but using 6 eggs; 9 eggs and 30 eggs.

4- When filling tanks with a capacity of 10 liters each, we spilled 0.25 liter on the ground.

- How much do we spill when we fill 20, 30 and 50 tanks of this kind?
- What is the number of tanks filled if you have spilled 15 liters?

5- • Calculate in cm^2 the areas of each of the triangles CAB, DAB, EAB and FAB then fill in the table:
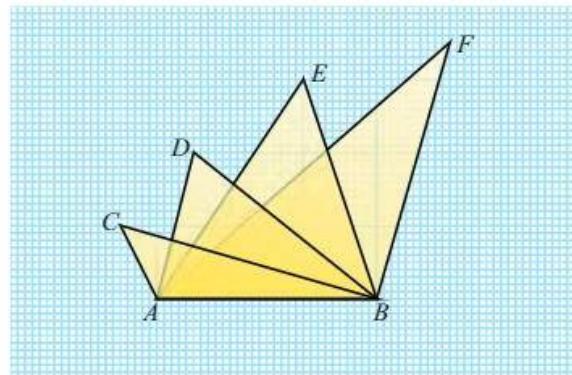
Triangles	CAB	DAB	EAB	FAB
Height				
Area				

- What is the area of a triangle MAB (not drawn) of height 14 cm?

6- Draw five rectangles that have a width of 4 cm with different lengths.
Calculate for each rectangle, its perimeter and.
Reproduce and complete the table:

Length of rectangle	....	....	....	....	....
Perimeter	....	....	....	....	....
Area	....	....	....	....	....

- Is there proportionality between:
  - The sequence of lengths perimeters?
  - The sequence of lengths areas?
- What is the perimeter of a rectangle of 118 cm in length and 4 cm in width?

143

7- A bus can transport 36 students. Calculate the number of buses needed to transport 144 students going on a trip.

The consumption of fuel by a bus is 0.22 liters per *km*. How much fuel will one bus need if the trip is 230-*km* long? How much fuel will all buses need?

The price of 20 liters of octane (fuel) is 13000 LL. Calculate how much the trip will cost.

8- To make jam, mother uses 2.7 *kg* of sugar for 3.5 *kg* of fruit.

1) Calculate:

- The quantity of sugar needed for 15 *kg* of fruit.
- The quantity of fruit corresponding to 20 *kg* of sugar.

2) Knowing that the mixture fruit-sugar loses 70% of its mass when boiled, what are the obtained quantities of jam in the two cases of the previous question?

Just for fun
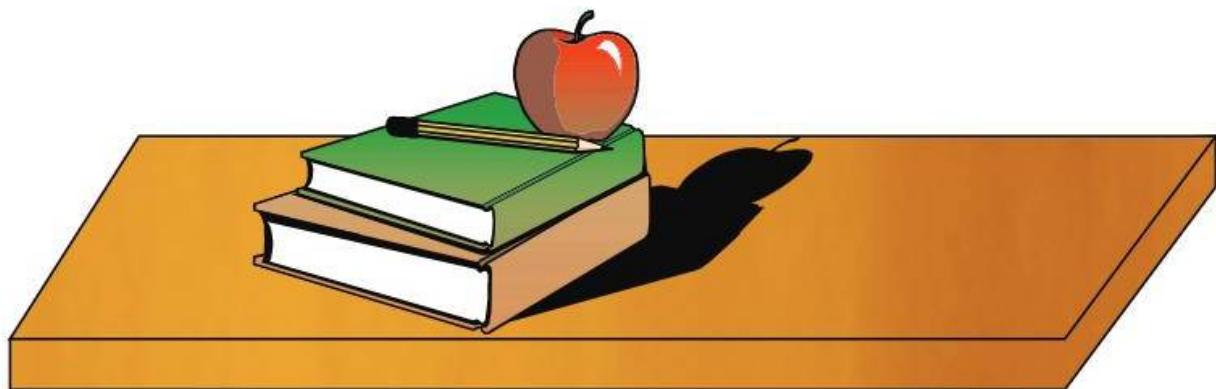
Place three different objects in the sun, knowing the height of the objects. Measure the length of the shadow of one of the objects.

Find without measuring the length of the two other shadows.

Statistics

Objectives

At the end of this chapter, I will be able to:
- [ ] read a circular diagram and translate the information into another type of representation.

Activities
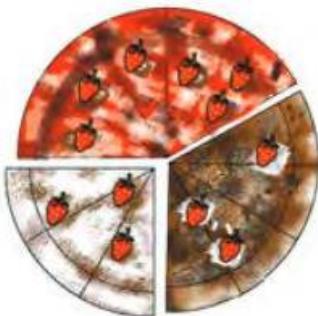
- We were 12 at Hadi's birthday, and the cake was divided into 12 equal pieces. Some of the pieces were cream flavored, others were strawberry flavored, and some were chocolate flavored. Observe the cake representation and find how many pieces of

- On another similar occasion where there were more people, we subdivided the cake so that each person would get a piece. What was the number of persons present according to the diagram given next?
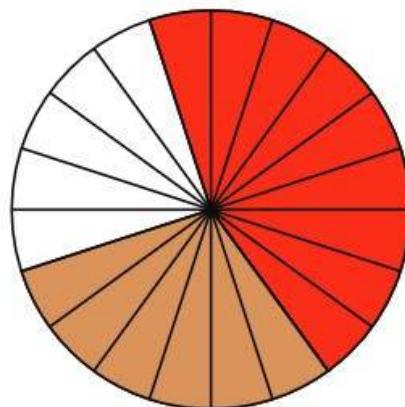
- Fill in the table with the correct items:

Cake type	cream-flavored	chocolate-flavored	strawberry-flavored
N° of items	.....	.....	.....
Percentage			
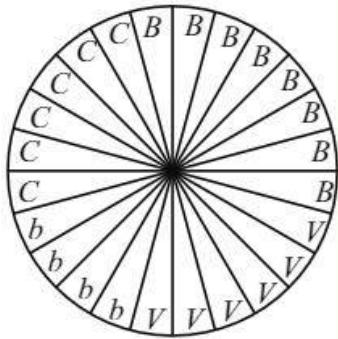
As is the case for the item table and the bar diagram, the circular diagram cut into sectors contains the given of a situation in one of the following forms:

a) Item - diagram:

The diagram given next is divided into 24 equal parts, concerns "24 individuals".

The signs B(bus), V(van) C(car) and b(bicycle) indicate that the "24 individuals" are classified into 4 categories according to the means of transportation used.

The number of items in each category is equal to the number of parts it contains. The circular diagram is translated to:

- an item table:

Means of transportation	Bus	Van	Car	Bicycle
N^o of items	8	6	6	4

- a bar diagram:
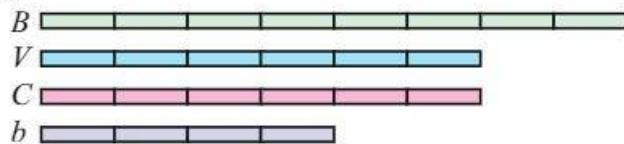
b) Circular diagram of percentages:

The circular diagram drawn next is relative to 400 members of a sport club. It is cut into three sectors: F (football), V (volleyball) and B (basketball).

The indication 56.25 % for sector football shows that:

$56.25 \times \dfrac{400}{100}$ or 225 members of this club practice football;

so it is for the sign 25 % on V (volley) showing that $25 \times \dfrac{400}{100}$

or 100 members of this club are volleyball players.

So the percentages inscribed on each sector allow to obtain the number of items relative to each activity from the total number of participants in this club, and to transform the circular diagram into an item table:
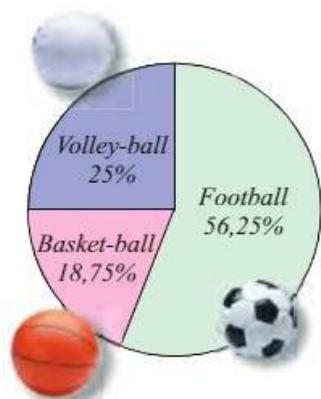
Sport activity	Football	Volleyball	Basketball
N^o of items	225	100	75
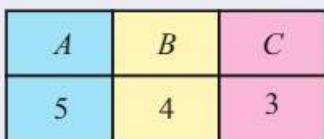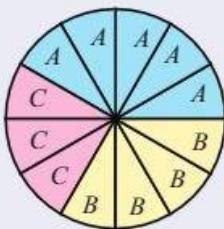
![Exercises icon] **Exercises**

1- Answer by "True" or "False":
The colored part of the disc represents the percent part inscribed below it:

25%

50%

30%

12.5%
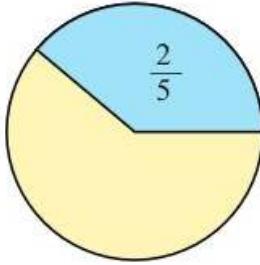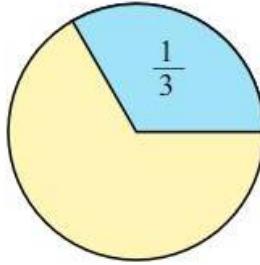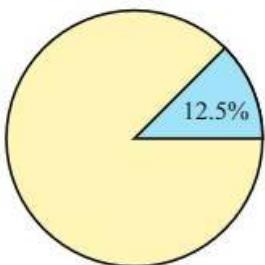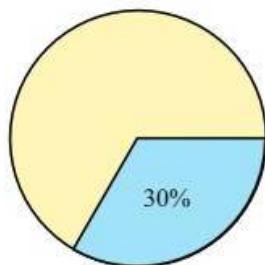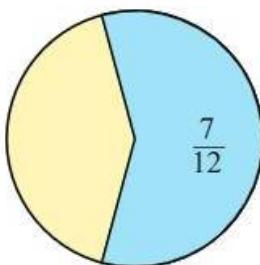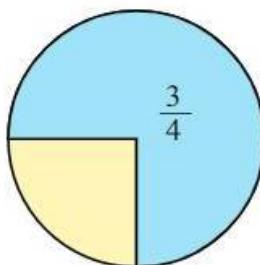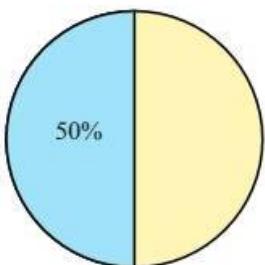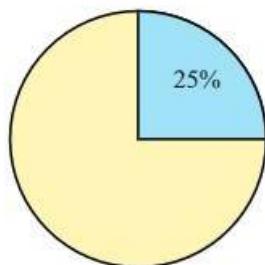
The striped part is equal to the fraction written above the disc.

$\dfrac{1}{4}$

$\dfrac{3}{8}$

$\dfrac{3}{4}$

$\dfrac{7}{12}$

$\dfrac{1}{3}$

$\dfrac{2}{5}$
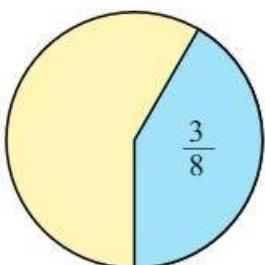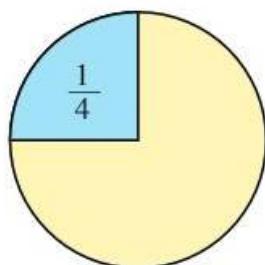
2- In a 25-student class, 10 are learning Spanish as a third language, 7 are learning German and 8 Italian.
Verify using calculations analogous to:

$$\frac{7}{25}=\frac{4\times7}{4\times25}=\frac{28}{100}$$

and using angles that the parts representing each of the three languages correspond to the calculated percentages.

Spanish

German

Italian
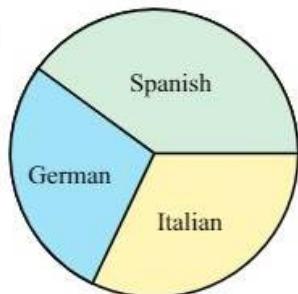
3- Find in each case the percentage represented by each colored part:
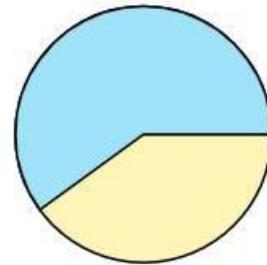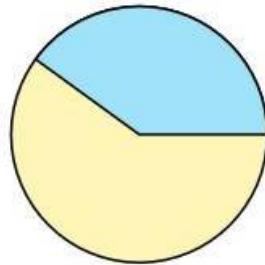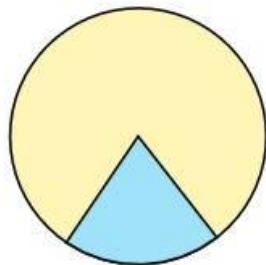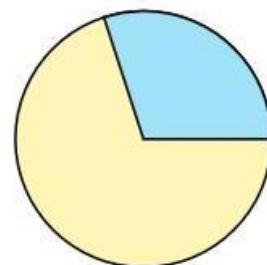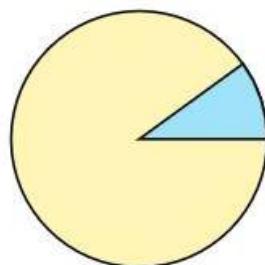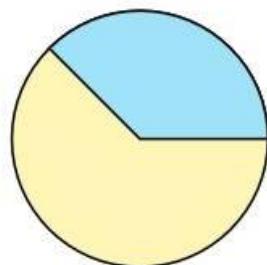
Self-evaluation

Do the two diagrams say the same?
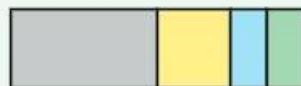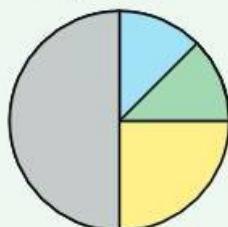
Problems

1- When taking out the preferences for a leisure place in the two sections of a 6th grade class, we obtained the following circular representations:
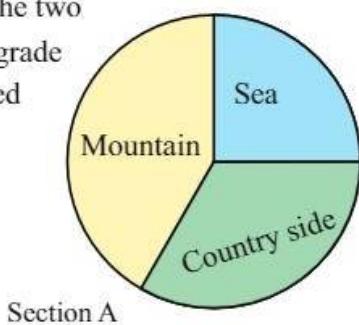
Section A

Section B

• In which of the two sections is the higher percentage of those preferring the "sea"?
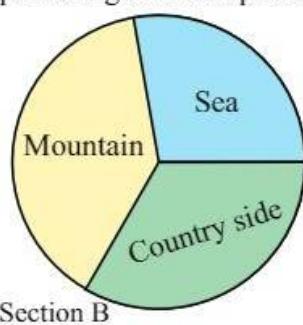
What can you say about the percentages of those who like to go to the country side?

2- The partition of the expenses of a certain family is given by the band diagram given below:

food ☐   electricity ☐
octane ☐   cloth ☐
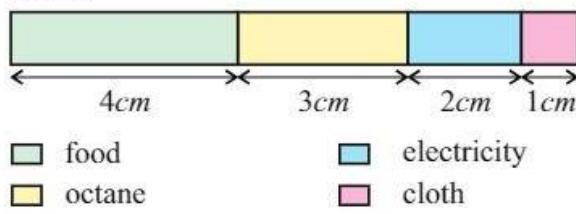
• Measure the length of each part of the band and determine the percentage it represents of the total length.

• Represent the given as percentages on the following circular diagram.
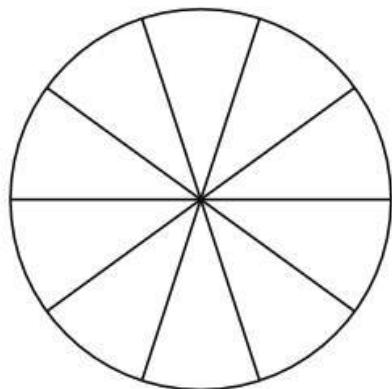
3- On a box of 48 crayons, we see the following representation:
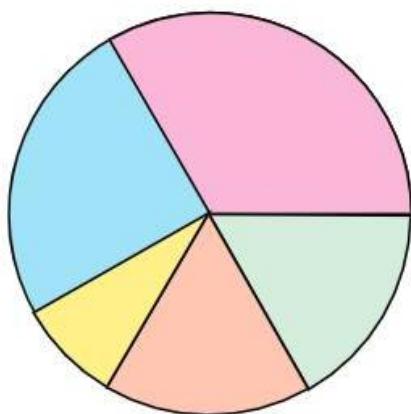
• What is the number of yellow crayons in this box?

• Complete the table:

Color	Yellow	Blue	Green	Orange	Red
N° of items	....	....	....	....	....

• Represent, this given in the form of a bar diagram.

• What will be the item table of another crayon box having the same representation but containing 720 crayons?

4- The circular diagram given below gives information on the consumption of different types of primary energies in a certain country:
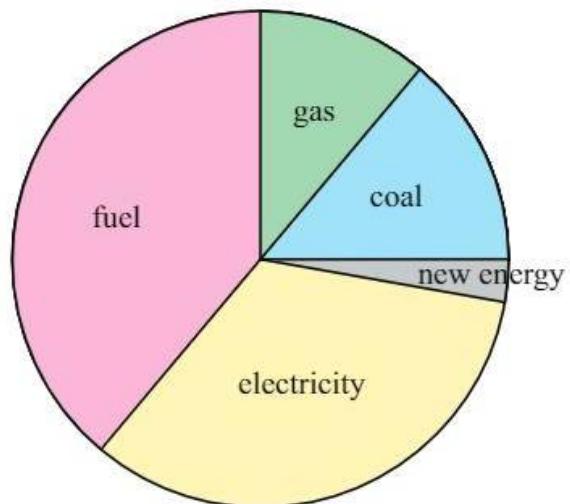
• Measure the angles of the different sectors.

• Calculate the fractions representing these sectors as parts of the disc.

• Transform the obtained fractions into percentages and fill in the following table:

Type of Energy	Coal	Fuel	Gas	Electricity
Percentage				

149
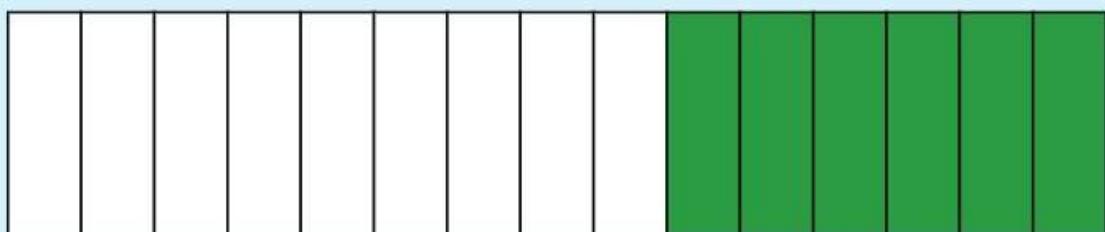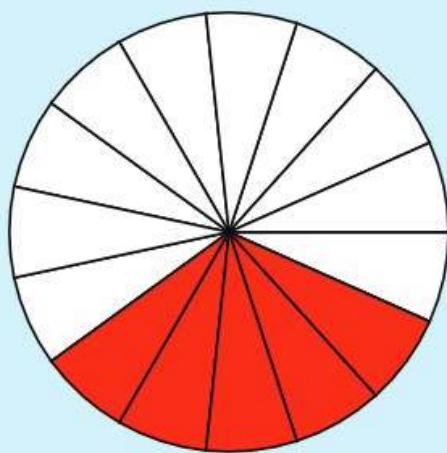
These two representations are for the percentages of students of a class whose names begin with the letter *A* or *S* or *M*. Complete them, and make an identical representation for the initials of the names of the students of your class.

# Addition and subtraction of signed numbers

24

Objectives

At the end of this chapter, I will be able to add and subtract many signed numbers.

Activities

Activity 1
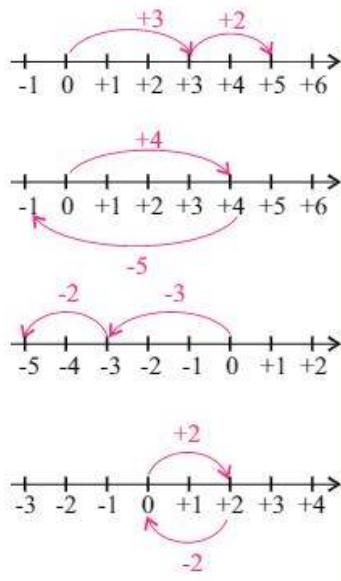
a) Reproduce the following graduated axis:
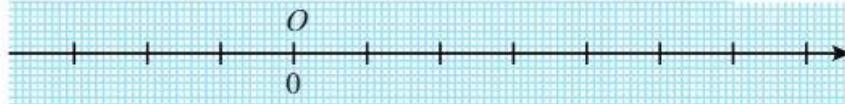
b) 4 ladybugs are moving on this axis:

The first starts from zero, moves 3 steps to the right, then 2 more steps.

Starting from zero, the second moves 4 steps to the right then goes back 5 steps.

Starting from zero, the third moves 3 steps to the left then 2 more steps.

The fourth one, also starting from zero, takes 2 steps to the right then 2 steps to the left.

What will the final position of each ladybird be?

Copy and complete:

$(+3) + (+2) = $; $(+4) + (-5) = $; $(-3) + (-2) = $; $(-2) + (+2) = $

c) Reproduce and complete the following table :

+	+7	-6	+16.14	-5.05
+3.5				
-10				

What number should we add to +15 to obtain +18?

What number should we add to +12 to obtain -10?

What number should we add to -10 to obtain -70?
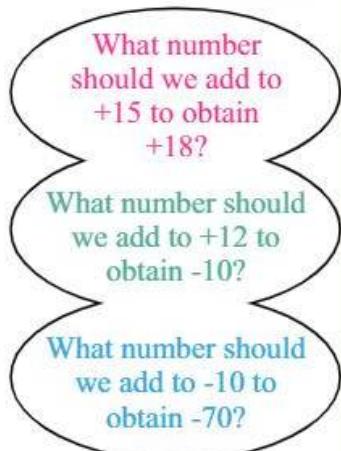
a) Inside a plane going from Beirut to Paris, there was a thermometer indicating 15° at the start of the flight.
The weather forecast was 18°C.
Calculate the difference between these two temperatures

b) At the altitude of 1500 m , the temperature was -10°C but inside the plane, the temperature was 12°C.

What is the difference between these two temperatures?

c) To Paris, the weather forecast was -7°C.
Find the difference between the temperature in Paris and that at the altitude of 1500 m.

d) Reproduce the table given below, then complete:

$(+18) - (+15) =$	$(+18) + (-15) =$	$(+15) - (+18) =$
$(-10) - (+12) =$	$(-10) + (-12) =$	$(+12) - (-10) =$
$(-7) - (-10) =$	$(-7) + (+10) =$	$(-10) + (-7) =$

Compare the results obtained in:
  the 1st and 2nd column;
  the 2nd and 3rd column.

e) Copy and complete:
  to subtract a signed number is to add its ....

Text

$(+1) + (+2) = +3.$
$(-1) + (-1) = -2.$

$-5 + (+2) = -3$
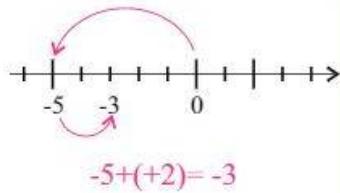
Addition of two signed numbers:

The sum of two signed numbers of the same sign is a number equal to the sum of their distances zero preceded by their common sign.

The sum of two signed numbers of different signs is a number equal to the difference of their distances from zero, preceded by the sign of the number that is farther from zero.

The sum of two opposite signed numbers is zero.

$+4 + (-4) = 0$
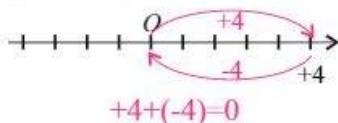
Subtraction of two signed numbers:

To subtract a signed number from another is to add its opposite to the other.

$(+3) - (-2) = (+3) + \text{opp} (-2) = (+3) + (+2).$
$(+3) - (+2) = (+3) + \text{opp} (+2) = (+3) + (-2).$

Focus

Adding two signed numbers *a* and *b*.

$(-2) + (-3)$

a and *b* are of the same sign

a and *b* are of different signs

$(+5) + (-2)$

$2 + 3 = 5$

we add their numerical parts

we subtract their numerical parts

$5 - 2 = 3$

-5

we precede the sum by their common sign

we precede their difference by the sign of the number farther from zero

+3

Subtracting two signed numbers *a* and *b*

$$a - b = a + opp(b)$$
$$(+5) - (-3) = (+5) + (+3) = + 8.$$

Exercises

1- Without doing any calculations, indicate in each of the following cases, the sign of the result: $(+5) + (+7)$
$(-5.2) + (-9)$
$(-15.5) + (+15.4)$
$(+4.9) + (-0.3)$
$(-6.73) + (+0.1).$

2- Calculate:

$(-3) + (+6) =$ $(+3) + (-6) =$
$(+3) + (+6) =$ $(-3) + (-6) =$

3- Calculate:

$(-2.1) + (-7.3) =$ $(-2.5) + (-0.3) =$
$(+27) + (-5.1) =$ $(-3.1) + (-1.1) =$
$(+2.5) + (+0.5) =$ $(-5) + 0 =$
$(-8) + (+0.9) =$ $(+7.3) + 0 =$

4-
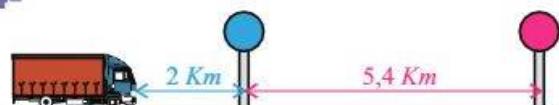
Calculate:

$(+6) + (-3) + (+5) =$
$(-7.2) + (+5.1) + (-5.1) =$
$(-17.42) + (+0.42) + (-7) =$
$(+37) + (-13) + (+4) =$
$(+12) + (+5) + (-12) + (+16) =$
$(-6) + (+7) + (-3) + (+18) =$
$(+2.5) + (-3.2) + (-2.5) + (+3) =$

then check your answers by means of your calculator.

5- Copy and complete:

$(+5) +$ .... $= 0$

$(+4.2) +$ .... $= 0$

$(-7.25) +$ .... $= 0$

$(-16.02) +$ .... $= 0.$

6- Copy and complete the following table:

a	b	a + b	opp(b)	a + opp(b)
+3	+5			
-72	+6			
0	-4			
+10	-5.2			
-5	-3.5			

7- Calculate:

$(-5) - (+7) =$ $(-3.1) - (-2.5) =$

$(-8) - (-12) =$ $(-5.4) - (+9.1) =$

$(+51) - (+7) =$ $(+3.2) - (+5.9) =$

$(+17.3) - (+6) =$ $(+2.5) - (-3.9) =$

$(+6.9) - (-0.3) =$ $(+2.5) - (+2.5) =$

8- Calculate:

$(-2.5) - (-3.7) + (-5.4) =$

$(+9.6) + (-2.4) - (-0.4) =$

$(-7.9) - (-6.4) - (-0.9) =$

$(-9.4) - (-6.1) - (-7.3) =$

then cheek your results with your calculator.

9- Calculate:

$(+9.12) + (+3.35) - (+7.81) + (+14.3) =$

$(-7.01) - (+17.3) - (+5.025) =$

$(+6.02) + (-11.17) - (-7.92) =$

$(+6.54) - (+7.92) - (-6.54) =$

10- Copy and complete the table:

x	y	x + y	x - y
+0.3	+2		
-10	-100		
0	+0.5		
+20	-20		

11- Calculate $a - b$; $b - a$; opp $a - b$ when:

1) $a = +6;$ $b = -3$

2) $a = -0.25;$ $b = +7.3$

3) $a = -9;$ $b = -3.4$

4) $a = +4.32;$ $b = +7.3$

5) $a = 0;$ $b = -8.1$

6) $a = +2.8;$ $b = 0.$

12- Copy and complete the following table:

a	b	c	b + c	a - (b + c)	a - b - c
+10	+20	+38			
-8	-9	-27			
+4	-12	+1			
+0.9	0	-10			

Compare the answers obtained in the 5th and 6th columns.

13- Copy and complete the table:

x	+5.1			-3.4	0
x + (-2.3)		+9.4			
x + (+7.1)			0		
x + (-0.25)					

1- Correct, if possible:

- (-5) and (+7) are two opposite signed numbers.
- The sum of two opposite numbers is zero.
- The opposite of a number is negative.
- The sum of two signed numbers of the same sign is always positive.
- Every integer is a signed number.

2- Calculate:

a) $(+4.25) + (-6.75) =$

$(+13.9) + (-13.9) =$

$0 + (+14.27) =$

$0 + (-2.12) =$

b) $(-15.1) - (-4.9) =$

$(+5.3) - (-3.2) =$

$(-20) - (+20) =$

$(-17.8) - (-17.8) =$

Problems

1- I think of a number then subtract 3.75 from it. I add 2 times 6.25; then I add 25 to the obtained result and the final number will be 100. What was my initial number?

2- In a bus, there are 35 passengers. At the first stop, 9 persons got off and 13 got on.

At the second stop 6 persons got off and 5 got on.
What is the number of passengers in the bus now?

3- a) Up on a mountain in Alaska, the temperature on a single day dropped from 6.7°C to -48.8°C.

Calculate the registered variation in temperature on that day.

b) In the South Pole, during two minutes, the temperature rose from -20°C to 7.2°C.

Calculate the corresponding variation in temperature.

155

4- a) On a given planet, the temperature drops from 350° in the day to -107°C at night.

Calculate this variation in temperature

(a day on the planet is equal to 58.5 day's on Earth).

b) On planet Venus, the temperature of the high atmosphere is -33°C and that of the ground is 480°C.

Calculate the corresponding difference in temperatures.

Just for fun

The magical triangle

Locate the numbers:

+4	;	+3	;	+2	;	+1	;	0	;
-1	;	-2	;	-3	;	-4	;		

In the circles of the figure given below in a way to get the sum of the numbers placed in the circles on each side of this arrangement equal to zero!
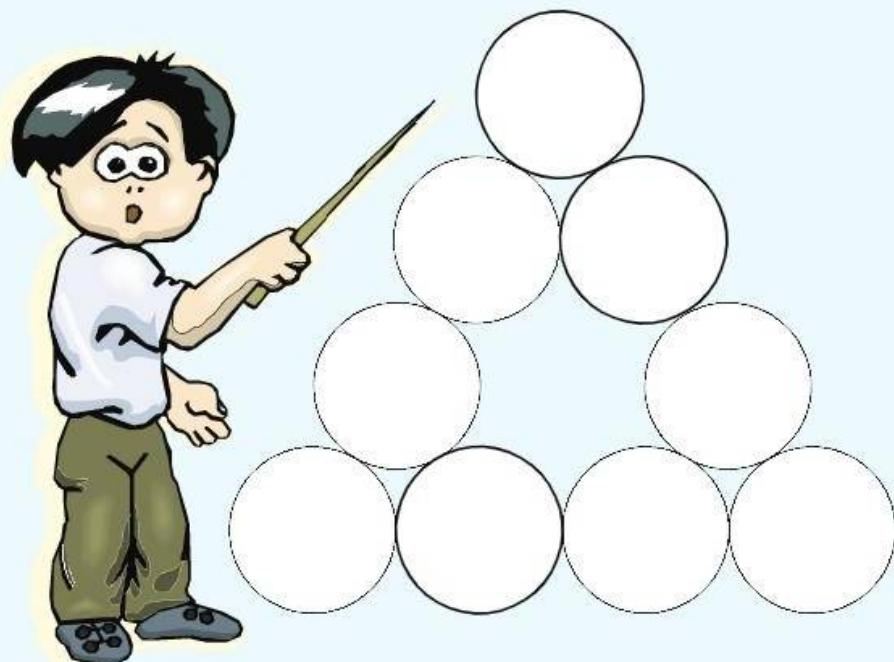
Volume (1)

Objectives

1- Calculate the volumes of solids: cube, rectangular prism, right circular cylinder, ball.
- recognize the volume of a solid.
- know the formulas giving the volumes of these solids.
- calculate volumes from formulas.

Activities

Activity 1

a)

water $1\ cm^3$

oil $1\ cm^3$
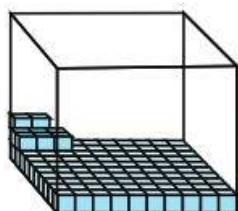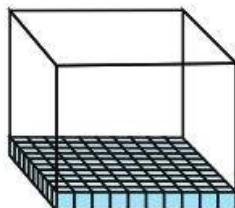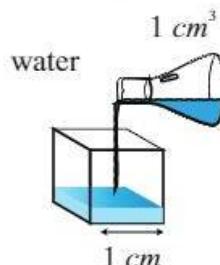
There is $1000\ mm^3$ in $1\ cm^3$
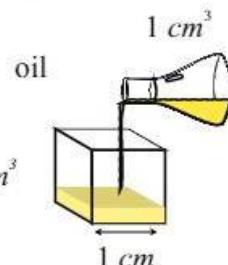
$1\ cm$ $1\ cm$

1 cubic centimeter ($1\ cm^3$) is the volume of a cube whose side's length is 1cm.

b) Construct a cube whose side's length is 1 dm.
How many cubes of side's length 1 cm are needed to fill this cube?

c) Copy and complete:
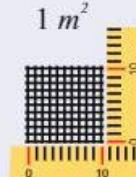
$1\ dm = \dots\ cm$	$1\ dm^2 = \dots\ cm^2$	$1\ dm^3 = \dots\ cm^3$
$1\ cm = \dots\ mm$	$1\ cm^2 = \dots\ mm^2$	$1\ cm^3 = \dots\ mm^3$
$1\ m = \dots\ cm$	$1\ m^2 = \dots\ cm^2$	$1\ m^3 = \dots\ cm^3$
$1\ m$	$1\ m^2$	$1\ m^3$
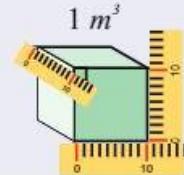
d) How many cubes of side's length 1*cm* are needed to fill a cube of - side's length 2*cm*?
- side's length 5*cm*? ....

a) What is the volume of a rectangular prism of length = 4 *cm*, of width = 3 *cm*, and height = 2 *cm*.

2 *cm* 3 *cm* 4 *cm*	3 *cm* 2 *cm* 4 *cm*	3 *cm* 4 *cm* 2 *cm*
Prism A	Prism B	Prism C

1 *cm*
1.5 *cm*
2 *cm*
Prism D
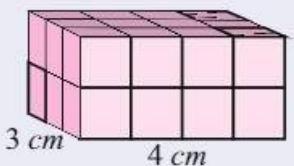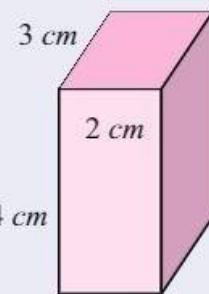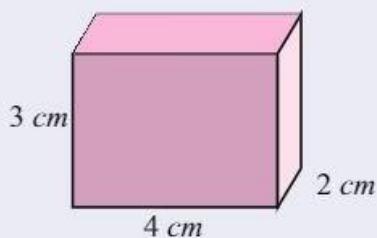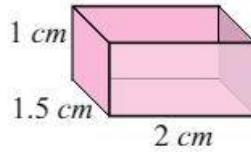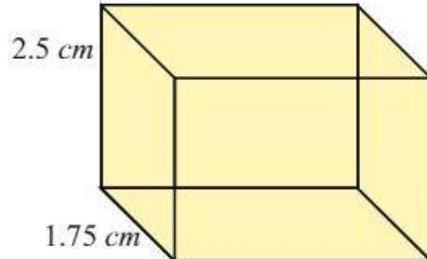
2.5 *cm*
1.75 *cm*
3.5 *cm*
Prism E

b) Copy the following table then complete it:

Rectangular Dimensions	A	B	C	D	E
length (ℓ) in *cm*					
width *w* in *cm*					
height (*h*) in *cm*					
volume (*V*) in *cm³*					

Use a calculator to check your answer.
Write the formula that enables you to calculate the volume of a rectangular prism.
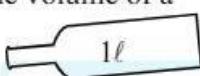
c) Calculate the area of the adjacent water aquarium.

What is its volume?

How many 1 liter bottles are needed to fill this aquarium?

1ℓ

Fish aquarium

25 *cm*
32 *cm*
5.6 *dm*
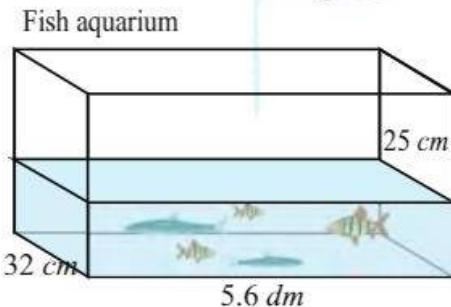
Note:
(m^3) is the volume of a cube whose side's length $= 1 cm$
$1\ell = 1\ dm^3$

1 litter = 1 dm^3

L $= 4\ cm$
$\ell = 3\ cm$
h $= 2\ cm$
$V = 4\ cm \times 3\ cm \times 2\ cm$
$= 24\ cm^3$
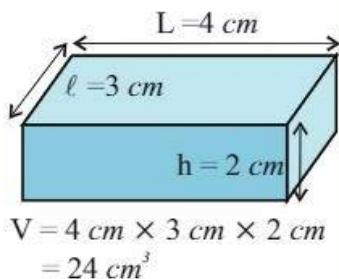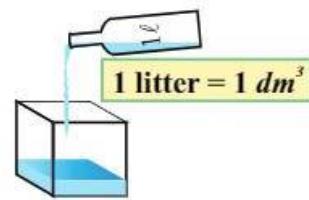
A- The lengths are measured by the following units:

| km; | hm; | dam; | m; | dm; | cm; | mm; |

The areas are measured by a real units:

| km^2; | hm^2; | dam^2; | m^2; | dm^2; | cm^2; | mm^2; |

We measure the volumes by the following units:

| km^3; | hm^3; | dam^3; | m^3; | dm^3; | cm^3; | mm^3; |

We can also use the capacity unit, the liter unit, as a volumetric unit.

B- Volume:

The volume of a cube which has equal sides is equivalent to the cube of its sides.
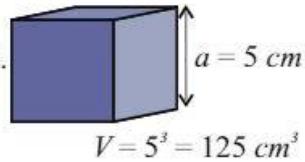
volume = side × side × side.

$a = 5\ cm$
$V = 5^3 = 125\ cm^3$

The volume of a rectangular prism is the product of its 3 dimensions lengths, ie, the product of its height and base.

Volume = $\underbrace{\text{length} \times \text{width}}_{\text{Area of base}} \times \text{height}$

• The table below shows the volumetric units:

mm^3	cm^3	dm^3	m^3	dam^3	hm^3	km^3
				7		

$\div 10^3$

$7\ dam^3 = 7 \times 1000\ m^3 = 7000 m^3$
$7\ dam^3 = 7000\ m^3$

$h\ell$	$da\ell$	ℓ	$d\ell$	$c\ell$	$m\ell$
				1	0

$10\ m\ell = 10 \div 10\ c\ell = 1\ c\ell.$
$10\ m\ell = 1\ c\ell.$

The volume of:

cube:
$V = a^3$
a

rectangular prism:
$V = L \times w \times h$
$V = \mathcal{A} \times h$
h
w
L

159

![Exercises character illustration]
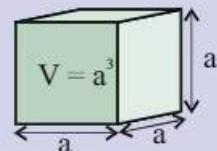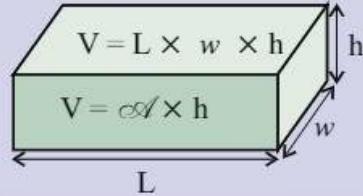
Exercises

1- Copy and complete:

$25\ m^3$ $= ....\ dm^3$
$750\ dm^3$ $= ....\ m^3$
$4.5\ dm^3$ $= ....\ cm^3$
$80\ cm^3$ $= ....\ dm^3$
$1248625\ mm^3$ $= ....\ dm^3$
$0.368\ m^3$ $= ....\ dm^3$.

2- What is the unit for measuring the capacity of:

milk can;
a mineral water bottle;
bath tub;
swimming pool;

3- Copy and complete:

$8\ d\ell\ 5\ c\ell = ...\ \ell.$
$2\ell\ 56\ d\ell = ...\ dm^3.$
$650\ell = ...\ h\ell = ...\ m^3.$
$1\ m^3 = ...\ \ell.$
$123648\ell = ...\ dam^3.$

4- Copy and complete the following table:

cube	A	B	C
side's length	0.6 m	52.04 dm	649.73 cm
volume			

Check the answers using the calculator.

5- Observe the figures:
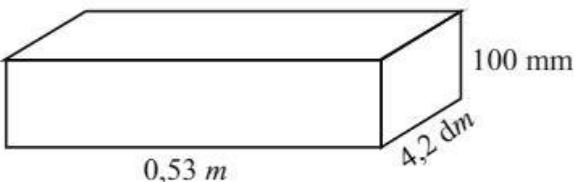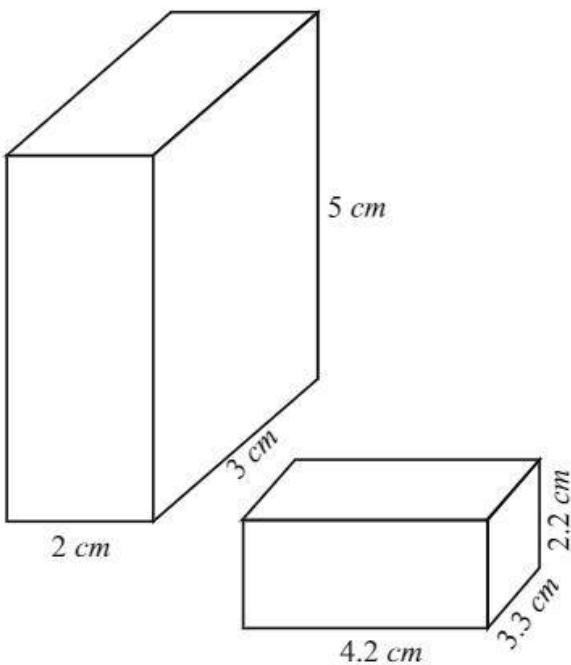
Measure the volume of each solid.
Use a calculator to check your answer.

![Self-evaluation character illustration]

Self-evaluation

1- Copy and complete:

$17\ dm^3 = ...\ cm^3$; $6.2\ h\ell = ...\ \ell$; $16\ dam^3 = ...\ell$;
$0.6\ cm^3 = ...\ \ell$; $3\ mm^3 = ...\ cm^3$; $9.4\ h\ell = ...\ m^3$.

2- What is in m^3 the volume of each solid:
 a) A cube with a side's length $= 12\ cm$?
 b) A rectangular prism whose dimensions are: $3.2\ m$; $53\ dm$; $100\ mm$?

**1-** 15 liters of medical solutions were prepared in a laboratory. They were emptied in bottles of capacity = 5 cm^3 each. In how many bottles was the solution poured?

2- Organize the following capacities in an ascending order:

3 $d\ell$; 15 $c\ell$; 0.05 dm^3 ; 40 $m\ell$;
18 cm^3 ; 0.26 litter; 1.9 $d\ell$.

3- The content of a juice jug was poured in 25 glasses, each of volume equal to 7 $c\ell$ and 5 $c\ell$ were left in it. What is the capacity of the jug?
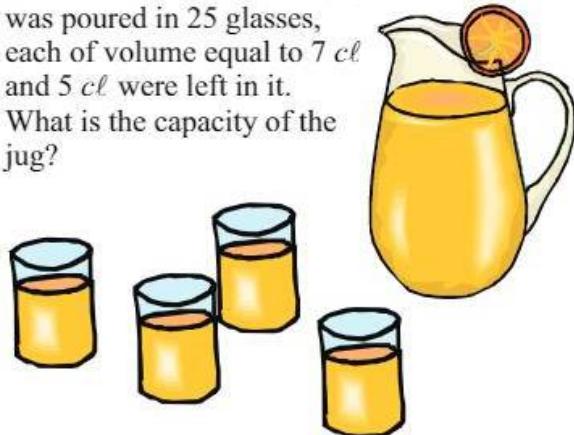
4- Given a rectangular area of 52 m, 35 m, we want to fill it with a layer of sand that is 10 cm thick. Knowing that the sand will be transported in a truck with a maximum capacity of 8 m^3,

a) find the volume of sand needed.

b) how many trips are needed for the truck to transport the volume of sand needed?

5- a) Observe the water aquarium below:
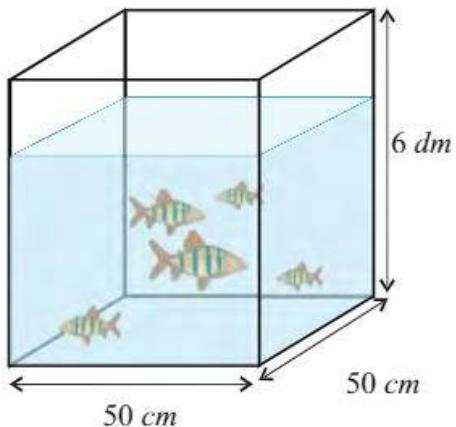
Calculate in liters the quantity of water it can contain.

b) If we want to fill $\dfrac{2}{3}$ of the aquarium, how many liters of water are needed?

c) If we empty 96 liters of water, what will the height of the remaining water be?

6- Due to a violent storm, water flowed on a rectangular porch whose width is 3.5 m and length 8.5 m. After the water was collected in an empty tank, its volume was 1 500 liters. Calculate in mm the height of the water reached on the porch before it was emptied in the tank.

A Candy Box

To construct a box that looks nearly like the one drawn to the right, follow the steps below:

a) Take an A_4 paper.

b) Fold it into 6 identical pieces and highlight them as in (figure ①).

c) Resold them according to *AB* and always towards the center (figure ②).

d) Fold the edges upwards and downwards (figure ③).

e) Fold it according to *CD* to the left (figure ④).

f) repeat the activity but this time in the opposite direction, ie, to the right of the paper (figure ⑤).

You will get what looks like the number 5 in Arabic.

Open the box by unfolding according to the enter opening.
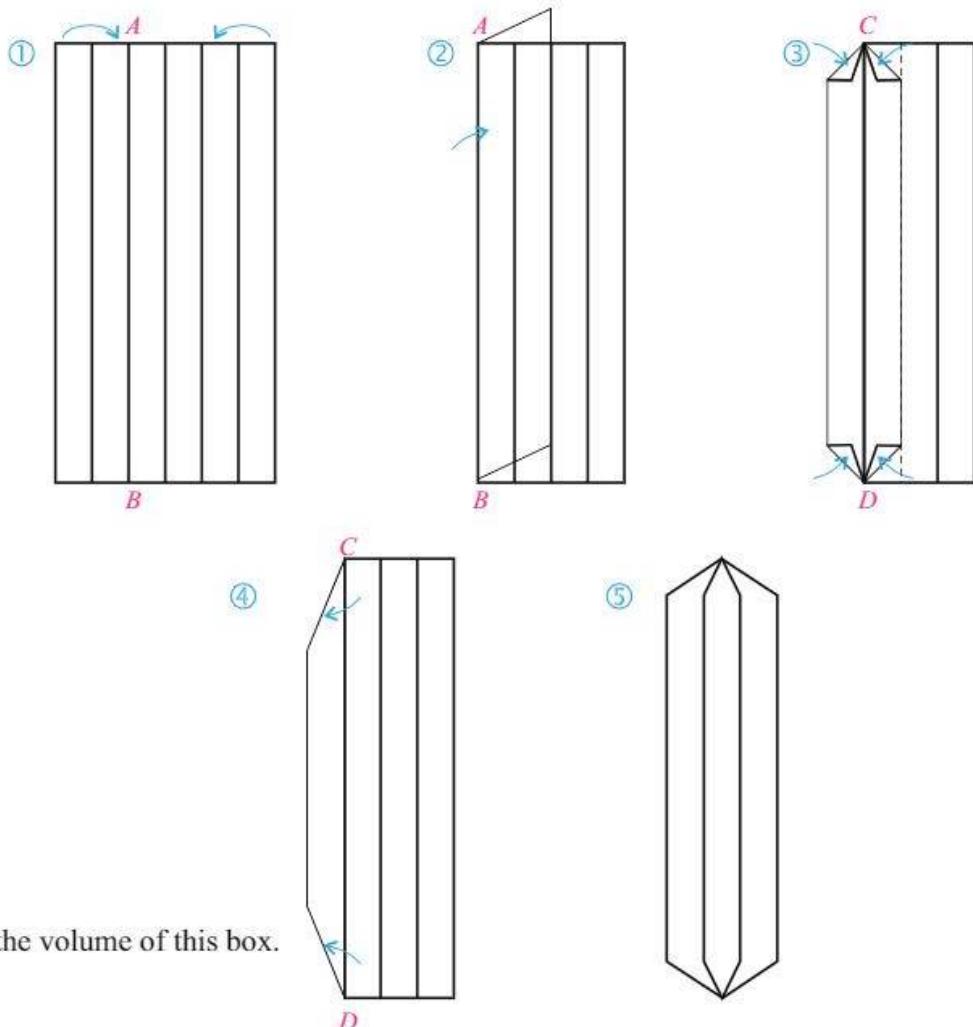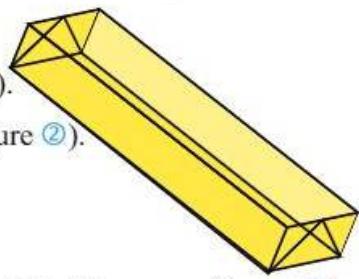
Find the volume of this box.

# Division of a duration by an integer

Objectives

At the end of this chapter, I will be able to:

- divide a duration by an integer inferior to ten;
- recognize the sub-multiples of a second.

Activities

A- A race began at $8\,hr\ 30\,min\ 19\,s$ and ended at $10\ hr\ 49\ min\ 52\ s$.

What was the duration of the race?
This race was divided into 5 steps of equal duration.
Calculate the duration of each of these steps.

B- The following durations indicate the time each participant needed to run 100 meters of the race.

Samir: $12s.25$ Farid: $11\ s\ \dfrac{2}{7}$

Kamal: $12\ s$ and 4 tenths   Nabil: $12s.3$

Salem: $13\ s\ \dfrac{3}{4}$ Raji: $13s$ and 45 hundredths.

1- Reproduce the table and put the durations according to their classification.

> To divide a duration by a natural number, you can directly divide but convert the remainder of hours to minutes and the remainder of minutes to seconds.
>
> **or** you can divide after converting all the durations to seconds.
>
> The hour is divided into 60 minutes and the minutes are divided into 60 seconds.

Rang	Name	Duration		
		seconds	$\dfrac{1}{10}$ of a second	$\dfrac{1}{100}$ of a second
1st				
2nd				
3rd				
4th				
5th				
6th				

2- Amin covered the same distance in 12 seconds. Can we put this value in the upper table? Why or why not?

3- Divide (8 hrs 30 min 32 s) by 4.

Text

A- To divide a duration by an integer inferior to 10:

$$7\ hr\ 28\ min\ 32\ s \div 5$$

1- **W**e divide the number of hours by this integer;

- we obtain the remainder in hours, then we convert it to minutes (by multiplying it by 60);

- we add it to the given number of minutes.

hour	minute	second
5		

	hour	minute	second
hour	7 ⁻5	28 ⊕	32
1	2 × 60 → 120		

2- Now, we divide the obtained number of minutes by this integer;

- the remainder obtained is converted to seconds;

- the number of seconds is added to that of the initial duration.

hour	minute	second
5		

	hour	minute	second
hour minute second		148 -10 48 -45	32 ⊕
1	29	3 × 60 → 180	

3- We divide the number of "seconds" by this integer;

The quotient of the duration 7 hr 28 min 32 s by 5 is 1 hr 29 min 42 s and $\frac{2}{5}$ of a second.

hour	minute	second
5		

	hour	minute	second
hour minute second			212 -20 12 - 10 2
1	29	42	

B- The other method consists of:

- Converting all the duration to seconds;

$$7 \ hr \ 28 \ min \ 32 \ s \div 5$$
$$7 \ hr \ 28 \ min \ 32s = 7 \times 3600$$
$$+28 \times 60 + 32 = 26912 \ s.$$

- Do the Euclidean division of the number in seconds by the given integer;

```
              5382 s
    5  |  26912 s
          - 25
            19
          - 15
            41
          - 40
            12
          - 10
             2
```

- Express the quotient obtained in seconds in the normal form: **hr - min - sec.**

$$5382 \ s \ \text{and} \ \frac{2}{5} = 1 \ hr \ 29 \ min \ 42 \ s \ \text{and} \ \frac{4}{10} \ \text{of a second.}$$

C- By subdividing the second into 10, 100 or 1000 equal parts, we obtain, respectively, the $\frac{1}{10}$, $\frac{1}{100}$ and $\frac{1}{1000}$ of the second. The sub-multiples of the second allow to express durations more accurately.

The duration $32 \ s$ and $\frac{5}{8}$ of a second:

$$= 32 \ s \ \text{and} \ \frac{5}{8} \times \frac{125}{125}$$

$$= 32 \ \text{second and} \ \frac{625}{1000} \ \text{of a second}$$

$$= 32.625 \ \text{seconds}$$

Focus

To divide a duration

   is to divide
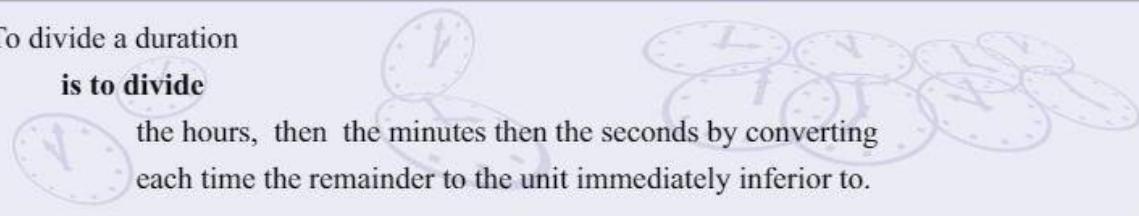
      the hours, then the minutes then the seconds by converting each time the remainder to the unit immediately inferior to.

Exercises

1- Divide directly:

$(5\ hr\ 12\ min\ 16\ s)\ \div\ 4$

$(33\ hr\ 15\ min\ 12\ s)\ \div\ 8$

$(7\ hr\ 16\ min\ 17\ s)\ \div\ 5$

$(14\ hr\ 50\ s)\ \div\ 10$

$(2\ hr\ 25\ min\ 32\ s)\ \div\ 7$

2- Convert then divide:

$(12\ hr\ 35\ min\ 11\ s)\ \div\ 5$

$(9\ hr\ 21\ min\ 22\ s)\ \div\ 4$

$(3\ hr\ 23\ min\ 45\ s)\ \div\ 9$

$(4\ hr\ 22\ min\ 56\ s)\ \div\ 6$

3- Choose the correct answer:

$65\ s\ \dfrac{1}{4}$	1 *hr* 5 *s* 14 tenths
	3 *min* and 5.4 *s*
	1 *min* and 5.25 *s*

$3\ s\ \dfrac{4}{5}$	15 *hr* and 4 tenths
	3.8 *s*
	3.4 *s*

4- A carpenter wants to fix the 4 borders of a square table in 5 *min* . In how much time can he fix a border?

5- It takes 4 *hr* 30 *min* for a worker to dig an 8*m* long water passage. What is the duration needed to dig1 *m*?

6- Complete:

| 2*hr* 30*min* 10*s* 5 *hr* 35 *s* .... | ×2 → .... 21*min* 3.2*s* .... 5 *hr* 7.16 *s* | ÷4 → .... 2 *hr* 51 *s* .... 15*min* 2.4 *s* |
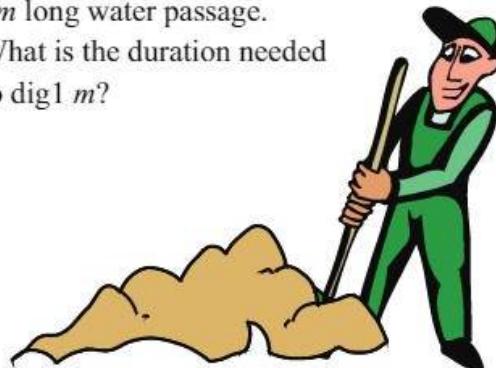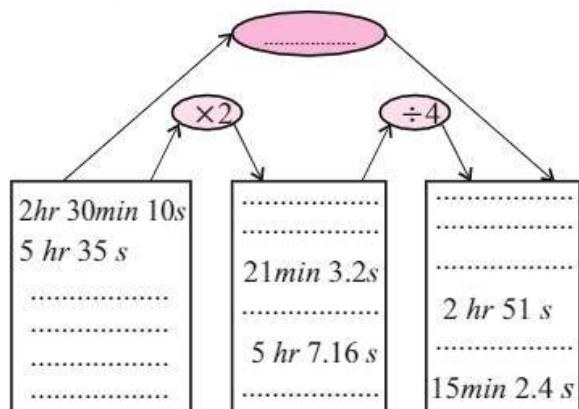
7- Use your calculator to do the conversions then the division of the duration:

1 day 12*hr* 15*min* 24.2*s* by 8
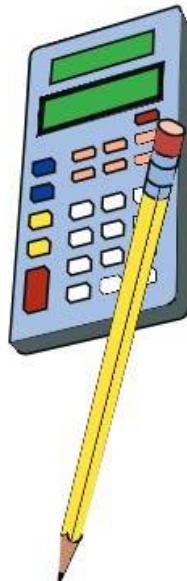
8- Calculate:

$(3min\ 33.4s+45s$ and $\dfrac{5}{10}$ of a second$)\ \div 6$

166

Discover and correct the (mistakes) Nawal made in the following division:

$$
\begin{array}{r}
1hr\ 5\ min\ 3\ s \\
6\ \overline{)\ 7hr\ 30min\ 22s} \\
-\ 6 \\
\overline{\quad 1hr} \\
31min \\
-\ 30 \\
\overline{\quad\ 1} \\
23s \\
-\ 18 \\
\overline{\quad\ 5}
\end{array}
$$
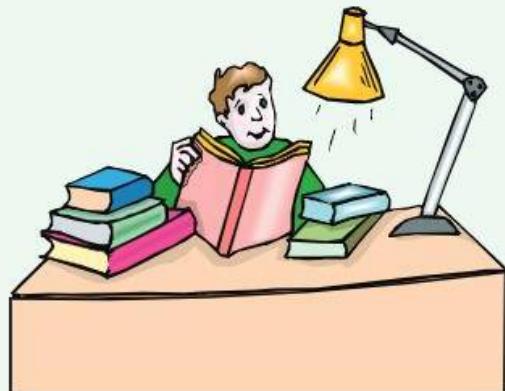
Problems

1- A tractor can plough 7 *dam²* in 6 *hr* 3*min* 25 *s*.
Calculate the duration that is necessary to plough 1 *dam²*.

2- Planet Mars takes 172*hr* 21*min* 41*s* to effect 7 complete rotations around itself. Calculate the duration of a single rotation.

3- During a car race, Habib started at 7 *hr* 32 *s* and $\frac{3}{10}$. After making 6 laps, the time was 7 *hr* 56*min* and 35.5*s*.
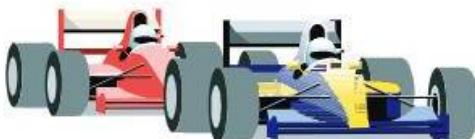
a) Calculate the time required for the 6 laps.
b) Calculate the duration of 1 lap.

4- A plane does 5 times the same trajectory in 42 *hr* 20 *min* 35 *s*.
If the speed of the plane is constant, calculate the duration of one trajectory.

5- A car moving at a constant speed from Sidon to Beirut covers this distance 6 times in 7 *hr* 30 *min*. Calculate the duration needed for this car to go from Beirut to Sidon.
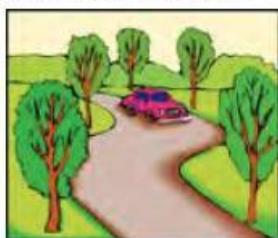
**6-** A tap empties a basin containing 7*hℓ* of water in 36*min* 32 *s*. How much time is needed to empty 1*hℓ*?

7- A secretary types 9 pages in 49 *min* 12.27 *s*. How long does it take her:

1) to type one page (supposing all pages are identical)?

2) to type 10 pages? 100 pages?

8- A boyscout runs 6 *km* in 55*min* 34*s*. What is the time needed to run 1 *km*? 15 *km*?

Just for fun

Everyday, Sami leaves his house riding a horse then comes back on foot. Knowing that the horse's speed is 25 *km/hr* and that Sami walks 5 *km*, and that each way has the same duration, calculate the daily trip.
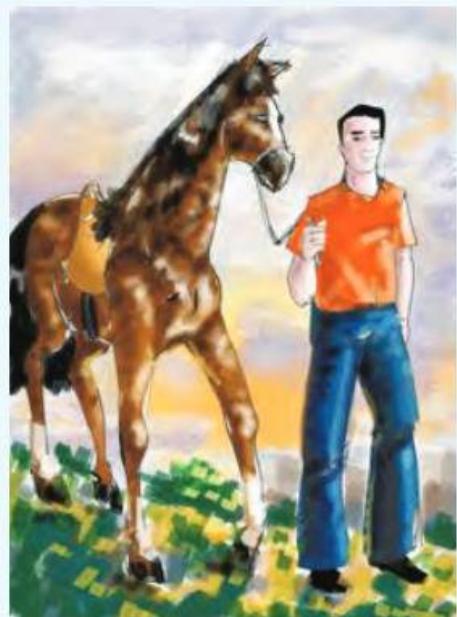
Volume (2)

Objectives

At the end of this chapter, I will be able to:

- Calculate volume of models in metric system.

Activities

Activity 1

Construct:

- a cylinder A of radius 11 *cm* and height 18 *cm*.
- two right rectangular prism whose side lenght are 20 and 18 *cm*.

Frame the volume of cylinder A following steps a), b) and c) as shown.

.......... < Volume of cylinder A <..........

then check the results using the following formula:

a)
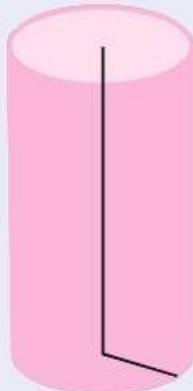
b)
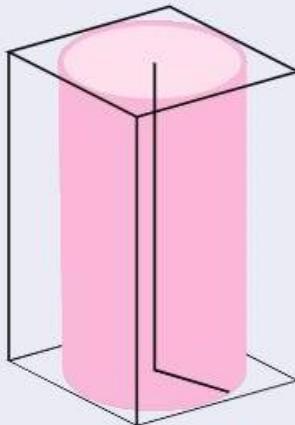
c)
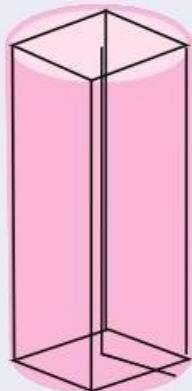
(use the calculator if needed).

**Volume of cylinder $= \pi r^2 h.$**

Observe the experiment below:
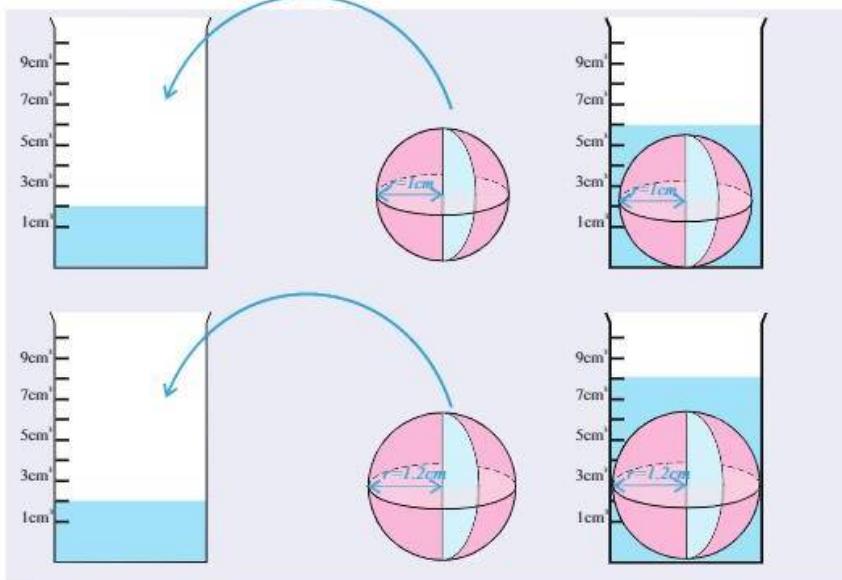
What is the volume of the yellow marble? The red one?

Check the two results obtained by using the following formula:

marble's volume equal to : $\frac{4}{3}\pi r^3$.

Text
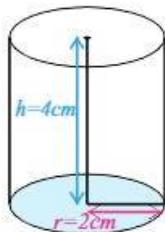
$V = 2\,cm \times 2\,cm \times 3.14 \times 4\,cm$

$V = 50.24\,cm^3$.
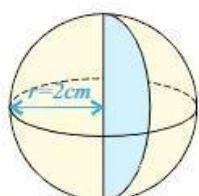
$V \approx \frac{4}{3} \times 3.14 \times 2\,cm \times 2\,cm \times 2\,cm$

$V \approx 33.49\,cm^3$.

Liquids and other soft substances take the shape of the container they are poured in. We talk about what this container contains or about its capacity.
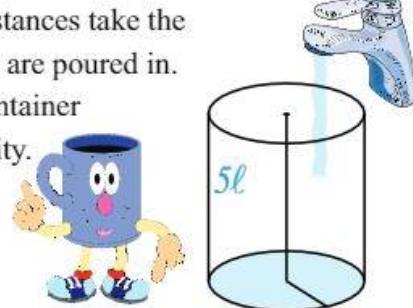
Volume:

• cylinder: is the product of its base × height:

> Volume of disc = radius × radius × π × height;
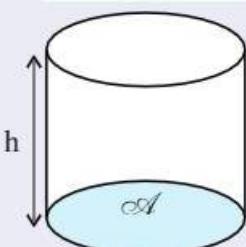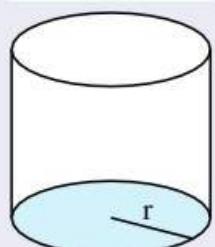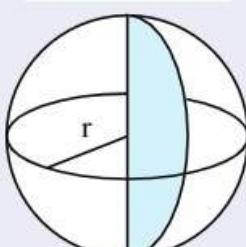
marble is:

> $\frac{4}{3}\,\pi$ × radius × radius × radius.

Volume	mm^3		cm^3			dm^3			m^3		dam^3		hm^3		km^3	
Capacity			$m\ell$	$c\ell$	$d\ell$	ℓ	$da\ell$	$h\ell$								

- Volume:

Cylinder:		Marbles
$V = \mathscr{A} \times h$	$V = \pi r^2 h$	$V = \dfrac{4}{3}\pi r^3$

V = volume
$\mathscr{A}$ = base's area
h = heigh

V = volume
$\pi = 3.14$
r = radius
h = height

V = volume
r = radius

Exercises

1- Convert to liters then to m^3:

    2.3 $h\ell$;   12 $da\ell$;

    2.4 hm^3 ;   456 $m\ell$;

    8 $d\ell$;   3.6 dm^3 ;

    6750 cm^3;   749 cm^3 .

2- In each of the following statements circle the most realistic answer:

Volume of water in a spoonful of soup is:
200 mm^3;   2 dm^3;   16 cm^3.

Volume of pine apple juice in a large bottle is:
3 ℓ;   4 dm^3;   1 ℓ.

Volume of air filling a room of dimensions:
4 m;   5 m;   and 3.5 m is:
45 dm^3;   45 m^3;   45000 mm^3.

Capacity of a cup of tea is:
10 dm^3;   1 cm^3;   1 m^3.

3- Find the volume of each of the cylinders represented below:

7 cm   2cm

17.3cm

6.32m   2dm

132mm
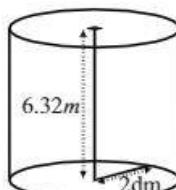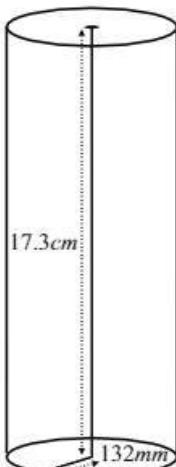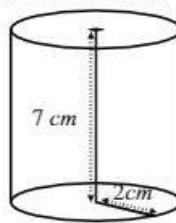
Then check your answers using the calculator.

4- Copy then complete the following table:

marbles	A	B	C
radius	17.32 cm	4.52 dm	0.05 cm
volume			

Check yours answers using the calculator.

The calculator can be used in Ex.: n° 5 - 6 - 7 and 8.

5- Copy then complete the following table:

cylinders	A	B	C
height	6 cm	10.3 m	7.8 cm
radius	4 cm	69 dm	4.7 m
volume			

6- What is the area of a cylinder's base in each of the following cases?
- volume 78 cm^3 and height 12 cm?
- volume 107.52 cm^3 and height 4.8 cm?
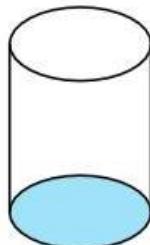
7- Copy then complete the following:

discs	A	B	C
radius	5 cm		
height	8 cm	4 m	
base's area			78.5 dm^2
volume in m^3		200.96 m^3	235.5 dm^3
volume in liters			

8- Calculate:

- volume of marble of radius 2.1 cm;

2.1 cm

- radius of marble whose volume is 904,32 cm^3.

?? cm
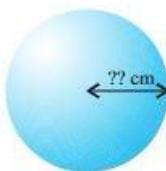
Self-evaluation

1- Copy and complete:

0.93 dam^3 = ... dal 1 hl = ...hm^3 1 cl = ...cm^3.

2- What is in m^3 the volume of:
- marble of radius 100 cm?
- disc of radius 32 cm and height 7.3 dm?

1- Which of the 2 boxes contains a larger mass of rice?

3.4 dm

20 cm

1.5 dm

100 mm

25 cm
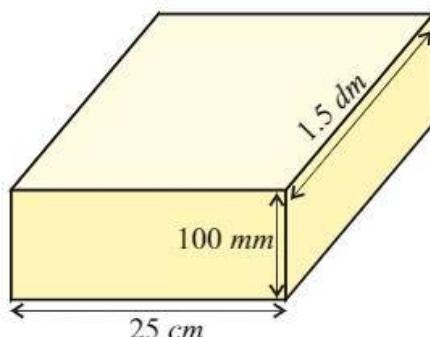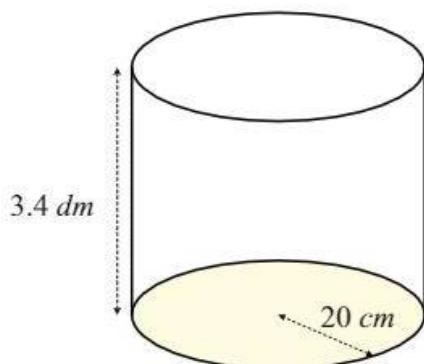
2- A baby bathtub's capacity is a double decaliter.

a) To fill this bathtub, how many jugs of 4.5ℓ of capacity do we need?

b) Calculate in liters the quantity of water remaining in the last jug after the bathtub is completely full.

3- The height of a cylinder shaped box is 12cm, and the diameter of its base is 7.5cm. Compare its capacity and that of a cubic box of side length = 9 cm.

4- A cylindrical plastic bag can hold 5 tennis balls each with a diameter of 7 cm.
a) Calculate the volume of one ball.
b) Find the height of the bag and its volume.

5- A cubic bowl is filled with water of side length equal to 10 cm.
a) We emptied its contents in a cylindrical tube of base radius 6 cm. How many cm did the height of water reach in the tube?
b) If we emptied the contents of the tube in glasses having a capacity each of 2 dℓ, how many glasses could we fill?

6- Sara used liquid dyes to draw on silk. She could get her favorite green dye by mixing 10 cm^3 of the blue dye with 40 cm^3 of her yellow dye.

• What is the volume of each of the blue dye and the yellow dye needed to get 125 cm^3 of the green dye?

7- A lady opened a tap water over a cylindrical jug of radius 27 cm and height 45 cm.
• Find the volume of water in the jug if its height reached 10 cm.
• What is the volume of water that should be added to the jug to become completely full?

Compare the volume of the earth to that of the moon, knowing that the radius of the earth is 3.8 times that of the moon. Taking into consideration that the earth and the moon are circular.
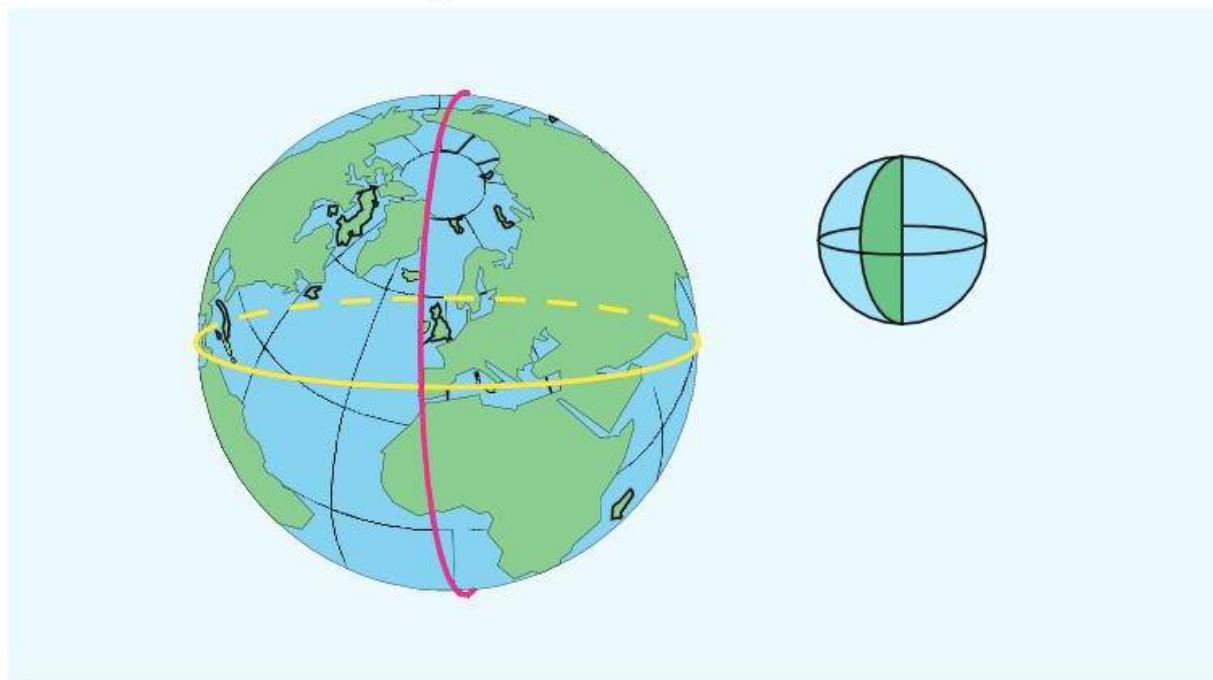
SELF-EVALUATION

Chapter 1   Order of operation

1) a) True.
   b) True.
   c) false $30 + 17 = 47$.
   d) False $15 + 18.16 = 33.16$.

2) a) $7 \times (2.6 + 3.4) = 42$.
   b) $23 \times (72 \div 9) = 184$.

3) a ⟍ ⟋ a
   b ⟋ ⟍ b
   c ⟋ ⟍ c
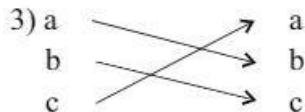
Chapter 2   Development of numbers

1) a) True   b) False.

2) - One hundred twenty-eight thousand, seven hundred fifty-four.

   - Thirteen million, seven hundred twenty-three thousand, four hundred fourteen.

   - Five billion, three hundred fifteen million, six hundred seventy-five thousand, four hundred one.

$128\ 754 = (1 \times 100\ 000) + (2 \times 10\ 000) + (8 \times 1000) + (7 \times 100) + (5 \times 10) + 4$.

$13\ 723\ 415 = (1 \times 10\ 000\ 000) + (3 \times 1000\ 000) + (7 \times 100\ 000) +$

$(2 \times 10\ 000) + (3 \times 1000) + (4 \times 100) + (1 \times 10) + 5$.

$5\ 315\ 675\ 401 = (5 \times 1\ 000\ 000\ 000) +$

$(3 \times 100\ 000\ 000) + (1 \times 10\ 000\ 000) +$

$(5 \times 1\ 000\ 000) + (6 \times 100\ 000)$

$+ (7 \times 10\ 000) + (5 \times 1000) + (4 \times 100) +$

Chapter 3   Lines and circles

Circle of center O and radius:

	1.8 cm	2 cm	3 cm	4 cm	5 cm
d	Zero	1	2	2	2
(D)	Zero	Zero	1	2	2
(D')	Zero	Zero	Zero	1	2

Chapter 4   L.C.M and G.C.D of two whole numbers

1) L.C.M (34 and 68) = 68
   G.C.D (34 and 68) = 34.
   L.C.M (16 and 24) = 48
   G.C.D (16 and 24) = 8.
   L.C.M (180 and 270) = 540
   G.C.D (180 and 270) = 90.
   L.C.M (45 and 90) = 90
   G.C.D (45 and 90) = 45.

2) The pairs that are prime with each other are:
   9 and 16;   36 and 37;   180 and 181.

Chapter 5   Patrons and solids

b) and d).

Chapter 6   Powers

1- $23.80013 \times 10^7 = 238\ 001\ 300$.
   $0.0002\ 435 \times 10^9 = 243\ 500$.

2- a) 9 is the **base** of the power 9^2.
   b) 3 is the **exponent** of the power 4^3.
   c) 6^2 is read as: "**6 square**".
   d) One million is a power of 10 in which the exponent is **6**.
   e) 2^5 is a power of **2**.

Chapter 7   Adjacent Angles –
Vertically Opposite Angles

1- Two vertically opp. angles: $P\widehat{O}C$ and $A\widehat{O}R$; $P\widehat{O}A$ and $K\widehat{O}R$.

- Two pairs of adjacent and complementary angles: $K\widehat{P}O$ and $O\widehat{P}A$; $P\widehat{A}O$ and $O\widehat{A}R$; $P\widehat{K}O$ and $O\widehat{K}R$; $A\widehat{R}O$ and $O\widehat{R}K$.

- Two pairs of adjacent and supplementary angles: $K\widehat{O}P$ and $P\widehat{O}A$; $K\widehat{O}R$ and $R\widehat{O}A$.

- Two complementary non-adjacent angles: $O\widehat{P}A$ and $O\widehat{A}R$.

- Two supplementary non-adjacent angles: $K\widehat{P}A$ and $K\widehat{R}A$.

2-

$u\widehat{O}x = v\widehat{O}t \ (V - O - A)$
$v\widehat{O}t = 35°$
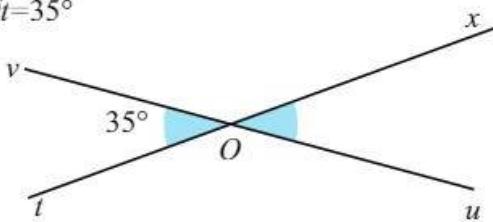
3-
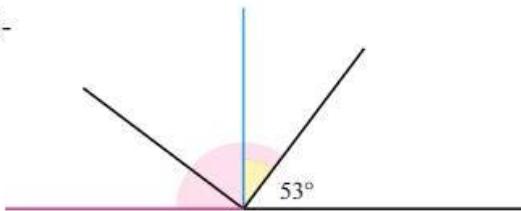
Chapter 8   Irreducible fractions

1- The irreducible fractions are:

$\dfrac{9}{7}$; $\dfrac{13}{15}$; $\dfrac{19}{3}$;

2- By using successive method:

$\dfrac{54}{42} = \dfrac{54÷2}{42÷2} = \dfrac{27}{21} = \dfrac{27÷3}{21÷3} = \dfrac{9}{7}$.

$\dfrac{42}{96} = \dfrac{42÷3}{96÷3} = \dfrac{14}{32} = \dfrac{14÷2}{32÷2} = \dfrac{7}{16}$.

$\dfrac{60}{195} = \dfrac{60÷5}{195÷5} = \dfrac{12}{39} = \dfrac{12÷3}{39÷3} = \dfrac{4}{13}$.

$\dfrac{72}{468} = \dfrac{72÷9}{468÷9} = \dfrac{8}{52} = \dfrac{8÷2}{52÷2} = \dfrac{4}{26} = \dfrac{4÷2}{26÷2} = \dfrac{2}{13}$.

$\dfrac{312}{26} = \dfrac{312÷2}{26÷2} = \dfrac{156}{13} = \dfrac{156÷13}{13÷13} = \dfrac{12}{1} = 12$.

$\dfrac{315}{280} = \dfrac{315÷5}{280÷5} = \dfrac{63}{56} = \dfrac{63÷7}{56÷7} = \dfrac{9}{8}$.

By using (G.C.D) method.:

- G.C.D (54 and 42) = 6 then

$$\dfrac{54}{42} = \dfrac{54÷6}{42÷6} = \dfrac{9}{7}.$$

- G.C.D (42 and 96) = 6 then

$$\dfrac{42}{96} = \dfrac{42÷6}{96÷6} = \dfrac{7}{16}.$$

- G.C.D (60 and 195) = 15 then

$$\dfrac{60}{195} = \dfrac{60÷15}{195÷15} = \dfrac{4}{13}.$$

- G.C.D (72 and 468) = 36 then

$$\dfrac{72}{468} = \dfrac{72÷36}{468÷36} = \dfrac{2}{13}.$$

- G.C.D (312 and 26) = 26 then

$$\dfrac{312}{26} = \dfrac{312÷26}{26÷26} = \dfrac{12}{1} = 12.$$

- G.C.D (315 and 280) = 35 then

$$\dfrac{315}{280} = \dfrac{315÷35}{280÷35} = \dfrac{9}{8}.$$

Chapter 9   Bisector of an angle

1-

1) In this fig. the semi st-line [OI) is not the bisector of the angle $A\widehat{O}B$.
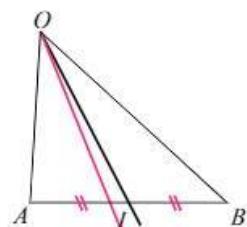
2) In this fig. the semi st-line [OI) is not the bisector of the angle $A\widehat{O}B$.
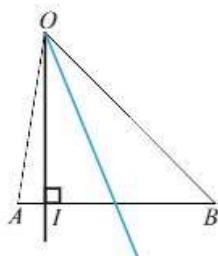
3) In this fig. the semi st-line [OI) is the bisector of the angle $A\widehat{O}B$. Since $A\widehat{O}B$ and $I\widehat{O}B$ are two adjacent equal angles.
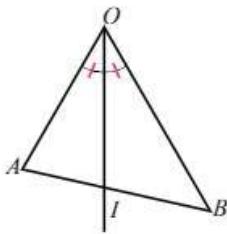
2- We have: $y\widehat{A}F = 48°$.

[At) bisector $y\widehat{A}t = t\widehat{A}F = \dfrac{y\widehat{A}F}{2} = \dfrac{48°}{2} = 24°.$
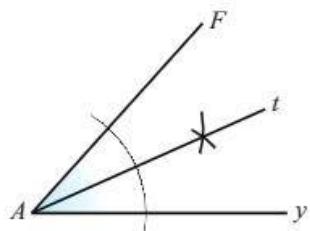
Chapter 10 Decimal fractions – Fractional writing of a decimal number

1- The decimal fractions are::

$$\dfrac{34}{25} ; \dfrac{12}{8} ; \dfrac{16}{4} ; \dfrac{63}{72}$$

$$\dfrac{34}{25} = \dfrac{34 \times 4}{25 \times 4} = \dfrac{136}{100} = 1.36.$$

$$\dfrac{12}{8} = \dfrac{3}{2} = \dfrac{3 \times 5}{2 \times 5} = \dfrac{15}{10} = 1.5.$$

$$\dfrac{16}{4} = 4.$$

$$\dfrac{63}{72} = \dfrac{7}{8} = \dfrac{7 \times 125}{8 \times 125} = \dfrac{875}{1\,000} = 0.875.$$

**2- a)** $0.7 = \dfrac{7}{10} = \dfrac{70}{100} = \dfrac{700}{1\,000} = \dfrac{70\,000}{100\,000}$

b) $5.36 = \dfrac{536}{100} = \dfrac{5\,360}{1\,000} = \dfrac{53\,600}{10\,000} = \dfrac{536\,000}{100\,000}$

c) $1.8 = \dfrac{18}{10} = \dfrac{9}{5}$; **d)** $0.52 = \dfrac{52}{100} = \dfrac{13}{25}$

e) $0.032 = \dfrac{32}{1000} = \dfrac{4}{125}$

Chapter 11 The perpendicular bisector of a segment

1- a) In this fig. d is not the perpendicular bisector of [AB]. [CD] passes through the mid-point of [AB] but it is not perpendicular to [AB] at its midpoint).
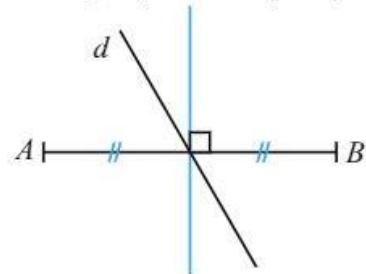
b) In this fig. d is not the perpendicular bisector of [AB]. Since ((d) perpendicular to [AB] but not at its midpoint).
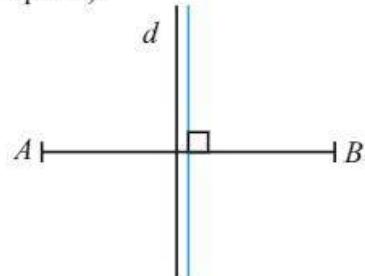
c) The st-line d is the perpendicular bisector of [AB] since d is perpendicular to [AB] at its midpoint.

2- We have: $SO=SF$ and **LF=LO.** [(LS) is the perpendicular of [OF]].

If a point is **equidistant from the extremities of a segment**, then this point belongs to the perpendicular bisector of this segment.

So, L and S are on the perpendicular bisector of [OF]

The perpendicular bisector of the segment [FO] is the st-line (LS).

Chapter 12   Triangles

1- a) • In triangle *PIE*, (*PT*) is the **median** issued from *P*.

• In triangle *PIE*, [*PI*] is the **bisector** of the **angle** $O\widehat{P}T$.

• The st-line (*PR*) is the **height** issued from *P* in the triangles *POD; PDI; PDT* and *PDE*.

• The st-line (*PR*) is the perpendicular bisector of triangle *POD*.

b) To obtain the center *A* of the circle circumscribed about triangle *POD*, we draw the perpendicular bisector of [*DP*] and of [*OP*].

2- a) 8 triangles.

b) • 4 right triangles: *VOT, VET, LOT* and *LET.*

• 3 isosceles triangles: *VOE, VOT* and *VET.*

• 1 equilateral triangle: *OLE.*

• 2 scalene triangles: *VOL* and *VEL.*

3)

$\widehat{T}$	$\widehat{R}$	$\widehat{I}$	Nature of triangle *TRI*
48°	54°	**78°**	Scalene
90°	24°	66°	Right triangle at *T*
60°	**60°**	60°	Equilateral
36°	**72°**	72°	Isosceles of vertex *T*
45°	45°	**90°**	Right isosceles (vertex *I*)
24°	106°	50°	Scalene

Chapter 13   Development of a decimal number in terms of powers of 10 and $\dfrac{1}{10}$.

| Twenty-five units and thirty-eight hundredth	25.38	$\frac{2538}{100}$	$25+\frac{38}{100}$	$25+\frac{3}{10}+\frac{8}{100}$
Fourty-six units and 5 tenth	46.5	$\frac{465}{10}$	$46+\frac{5}{10}$	$46+\frac{5}{10}$
Fourty-five hundredth	0.45	$\frac{45}{100}$	$0+\frac{45}{100}$	$0+\frac{4}{10}+\frac{5}{100}$
Three units and sixty seven thousandth	3.067	$\frac{3067}{1000}$	$3+\frac{67}{1000}$	$3+\frac{6}{100}+\frac{7}{1000}$
Eight units and fifty-seven thousandth	8.057	$\frac{8057}{1000}$	$8+\frac{57}{1000}$	$8+\frac{5}{100}+\frac{7}{1000}$
Six hundred fifteen thousandth	0.615	$\frac{615}{1000}$	$0+\frac{615}{1000}$	$0+\frac{6}{10}+\frac{1}{100}+\frac{5}{1000}$

2- a)

Number	Approximation to the units	Rounding to the unit
15.085	15	15
42.543	42	43
0.801	0	1
24.706	24	25

Number	Approximate to the tenth	Rounding to the tenth
15.085	15.0	15.1
42.543	42.5	42.5
0.801	0.8	0.8
24.706	24.7	24.7

Number	Approximation to the hundredth	Rounding to the hundredth
15.085	15.08	15.09
42.543	42.54	42.54
0.801	0.80	0.80
24.706	24.70	24.71

b) The numbers that have equal approximation to the tenth and rounding to the tenth are 42.543; 0.801 and 24.706.

Chapter 14   Cental symmetric
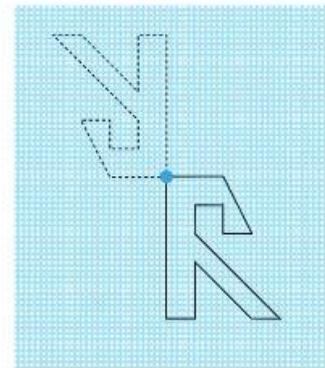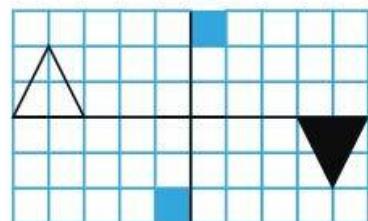
Chapter 15　Calculating areas

1) The area of the parallelogram is: 46.75 cm^2.

2) The measure of the height from R relative to [LM] is 3 cm.

Chapter 16　Calculation on literal expression

1) a) $7(3) + 5 = 21 + 5 = 26$ True.

b) False; $2(3 + 10) = 2(13) = 26$ not 30.

c) False; 17 taller than x is x + 17.

d) False; 6 more than double 7 all divided by 2 is $(2(7) + 6) \div 2$.

2)

a	b	c	a+(b+c)	a×b×c
21	11	5	37	1155
9.6	7.4	2	19	142.08
3	2.5	1	6.5	7.5

a×(b+c)	a+(b-c)	(a×b)+(a×c)
336	27	336
90.24	15	90.24
10.5	4.5	10.5

Chapter 17　Signed numbers

1- a)
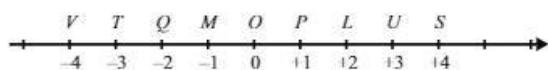
b) -1 abscissa of M.　　-3 abscissa of I.

-2 abscissa of R.　　-4 abscissa of N.

2)

a	opp(a)	distance from a to the origin	distance from opp(a) to the origin
+5	-5	5 cm	5 cm
-3.2	-3.2	3.2 cm	3.2 cm

Chapter 18　Comparison of signed number

1) $-7.2 > -7.5$　　　　$-16.03 < +13.06$

$+2.25 < +2.35$　　　$+5.1 > -19.49$.

2) $-6.33 < -6.3 < -6.03 < +6.03 < +6.33 < +7.1 < +7.17$.

3) all the negative numbers:

-10; -9; -8; -7; -6; -5; -4; -3; -2; -1; zero.

Chapter 19　Multiplying and dividing fractions

1) $\dfrac{10}{11} \times \dfrac{5}{11} = \dfrac{50}{121}$.

2) $\dfrac{17}{4} \times \dfrac{17}{3} = \dfrac{17^2}{12} = \dfrac{289}{12}$.

3) $\dfrac{3}{13} \times (\dfrac{3}{4} - \dfrac{2}{3}) = \dfrac{3}{13} \times \dfrac{3}{4} - \dfrac{3}{13} \times \dfrac{2}{3}$.

4) $2\dfrac{1}{3} \times 3\dfrac{1}{2} = \dfrac{7}{3} \times \dfrac{7}{2} = \dfrac{14}{6} = 2\dfrac{1}{3}$.

5) $35\dfrac{2}{7} \div 7\dfrac{1}{2} = \dfrac{247}{7} \div \dfrac{15}{2} = \dfrac{247}{7} \times \dfrac{2}{15} = \dfrac{494}{105} = 4\dfrac{74}{105}$.

Chapter 20　Quotient and ratio

1) The approximate value of the quotient of the division of two numbers: 123467 and 532.123 to the nearest 0.001 in excess is: 2320.079.

2) If the ratio of the area covered by water on earth to the area of the land is equal to: $\dfrac{71}{29}$ this means that the area of water is greater since 71 is greater than 29.

Chapter 21　Percentage

1) True - false - true - true.

2) Rima's pants are more expensive.

Chapter 22   Proportionality

1)

×5	0.5	0.1	2.5
	0.1	0.02	0.5

×0.2	0.65	1.2
	0.13	0.13

2)

2	10
7	35

1	1.2
20	24

30	6
0.4	0.08

11	4
33	12

3)

| 0.1	2	5	10
0.7	14	35	70

Chapter 23   Statistics

Yes, they are the same.
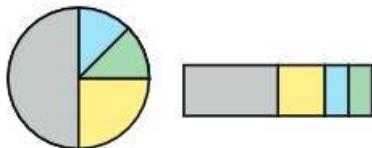
Chapter 24   Addition and subtraction of signed number

1) • (-5) and (+5) are two signed numbers symm. to each other.

• True.

• False, the opp. of a positive number is negative.

• False, the sum of 2 signed numbers of the same sign is the same sign as the numbers.

2) a) $(+4.25) + (-6.75) = -2.5$
$(+13.9) + (-13.9) = 0$
$0 + (+14.27) = + 14.27$
$0 + (-2.12) = -2.12$

b) $(-15.1) - (-4.9) = -10.2$
$(+5.3) - (-3.2) = +8.5$
$(-20) - (+20) = - 40$
$(-17.8) - (-17.8) = 0.$

Chapter 25   Volume (1)

1) $17\ dm^3 = 17000\ cm^3$
$6.2\ hl = 620\ell$
$16\ dm^3 = 16000000\ell$
$0.6\ cm^3 = 0.0006\ \ell$
$3\ mm^3 = 3000\ cm^3$
$9.4\ hl = 0.94\ m^3.$

2) a) $1728\ cm^3 = 0.001728\ m^3.$
b) $1.696\ m^3.$

Chapter 26   Division a duration by an integer

$7\ hr\ 30\ min\ 22\ second \div 6 = 1\ hr\ 15\ min$
$3\ second\ and\ \dfrac{4}{6}\ of\ a\ second.$

Chapter 27   Volume (2)

1) $0.93\ dm^3 = 9300\ da\ell.$
$1\ h\ell = 0.0000001\ hm^3.$
$1\ c\ell = 10\ cm^3.$

2) $2.355\ m^3.$
$23.472128\ m^3.$

Notes

116